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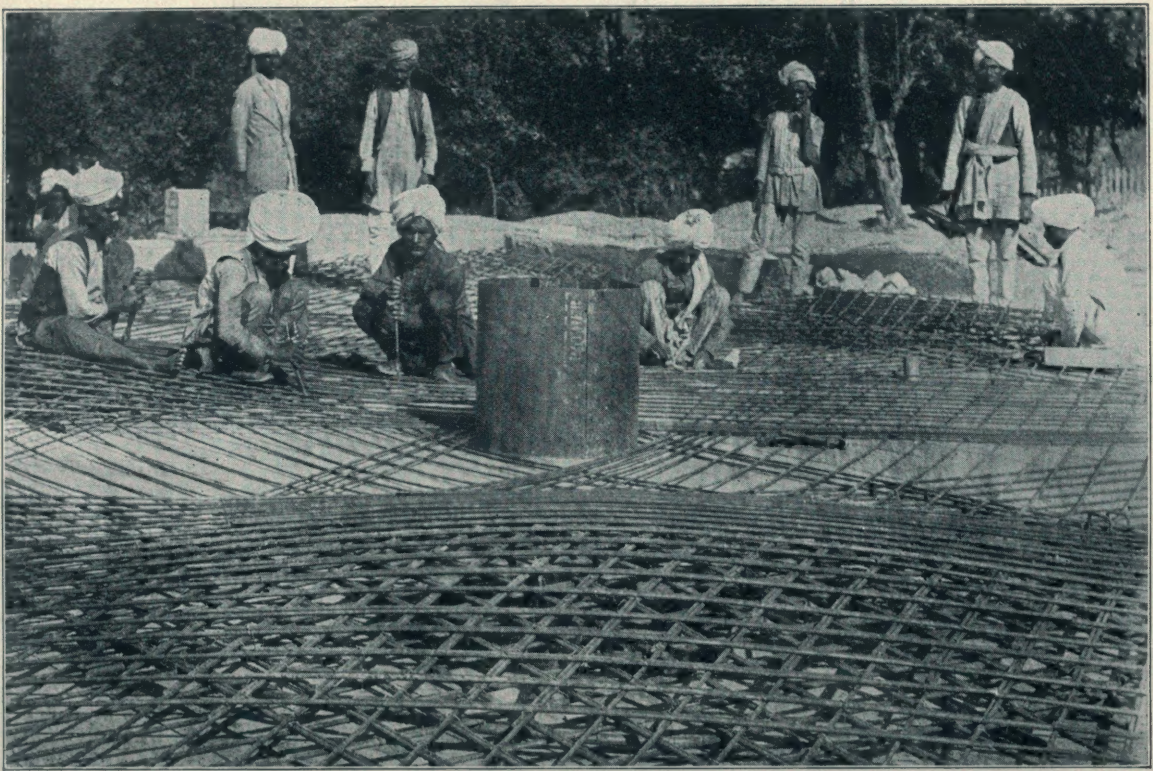

GARRISSON BAKERY FOR BRITISH GOVERNMENT, JULUNDER DISTRICT, DALHOUSIE, INDIA
Native laborers placing reinforcement steel—Mushroom System

ELASTICITY AND STRENGTH

OF

MATERIALS USED IN ENGINEERING CONSTRUCTION

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SECTION I

BY

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TO THE MEMORY OF THE PIONEER ENGINEER MATHEMATICIAN, HENRY TURNER EDDY, THIS WORK IS DEDICATED AS AN APPRECIATION OF THE VALUE OF HIS LIFE WORK IN THE ADVANCEMENT OF THE SCIENCE OF STRUCTURAL ENGINEERING.

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PREFACE

Large sums are yearly wasted in providing for the supposed injurious effect of moderate range of stress which really improves metal and at the same time heavy loss is suffered by failure to exercise practical judgment in the proportion and selection of material to resist shock in cases where the stress may at times exceed the natural range of elasticity of the material employed.

Section I is intended to demonstrate fundamental principles upon which rational analysis may be founded and to discuss the properties of materials commonly used from the standpoint of the elastician so that the student may apply the analytical methods of the second section understandingly and in a scientific manner.

A development of the relations of residual to elastic strain sufficient to account for elastic phenomena not understood heretofore is here presented for the first time and the error resulting from general misunderstanding of these relations is discussed.

That there is but one independent modulus of elasticity for the resistance of a homogeneous isotropic solid is here demonstrated for the first time and the relative value of the coefficients for such a body and their modification by residual strain are investigated.

The geometric relations of the curves of displacement and equipotential to the curves of principal stress and greatest shear are set in an exact light and the relations of these curves to the slip lines as the yield point is passed are developed.

Finally, stress and strain are discussed from the thermo dynamic standpoint and from the relations of the kinetic and static modulii the value of the potential of strain is developed in a simple manner.

Uncertainty as to stress analysis leads to waste or weakness, frequently to both.

Section II will be devoted to the elimination of such uncertainty for example, by the stress analysis of torsion, its part in column strength and failure will be developed, and the waste and weakness in common types will be disclosed.

Uncertainty with reference to the relative of laws of summation, of shear and moment has led to waste in construction of continuous trusses where simple trusses are stiffer and more economical, to waste and weakness in distribution of rivets in railway floor beams, and to unscientific and uneconomical types of webbing.

In reinforced concrete the common theory is based upon a fundamental misconception as to the operation of shear resulting in diverse formulas for shear and punching shear, to account for one and the same kind of shear strain. In formulas for plates the action of shear stress is irrationally attributed to the tension in concrete after the concrete is broken in tension!!

And finally the reversal of fundamental relations of states of stress demonstrated in Natural Philosophy by Thomson and Tait, appears in the analysis of plates by a Joint Committee Member.

C. A. P. Turner,
October, 1922.

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CHAPTER I.

1. All material used in construction, no matter how rigid it may appear when subjected to any load or applied force, suffers some change in dimension or shape. If this change in shape is linear it is called DEFORMATION; if angular DISTORTION. When the deformation or distortion does not exceed a certain amount, the material recovers its original form upon the removal of the load and its deportment is said to be ELASTIC.

The geometric relations between applied forces and the elastic changes in shape/which they produce is studied under the title of ELASTICITY.

The proper apportionment of material to resist given loads safely without dangerous or injurious distortion is known as the science of STRENGTH OF MATERIALS.

In abstract mechanics, for convenience, bodies are treated as though perfectly rigid and forces although actually distributed are considered as applied at material points; their direction indicated by lines and their relative intensity or magnitude by the length of the line laid off upon a convenient scale.

In investigation of elastic resistance, the forces to be considered are the distributed elastic reactions which the internal deformation or distortions oppose to the applied load or forces. Accordingly analysis of strength deals with the area of distribution of this reaction and in particular with the determination of the location and magnitude of the greatest intensity of the resisting forces per unit of area which, like the weakest link in the chain, determines the strength of the whole.

2. **Fiber Stress.**—As the unit change in shape (strain) and hence its reaction (stress) may vary in the material from point to point, the unit of elementary area considered must be very small. This unit is termed a fiber by the engineer though in mathematical theory it is treated as an infinitesimal area; $dx\ dy$: $dx\ dz$ referred to the orthogonal axes of XYZ of the Cartesian system of coordinates.

3. **Stress** in mechanics is the term used specifically to denote the relative intensity of the force brought to bear upon a given fiber or the equal and opposite reaction which the deformed fiber opposes to the given applied force in maintaining equilibrium. When stated

in units of force per unit of area, such as pounds per square inch or kilograms per square centimeter, it is a UNIT STRESS expressed in the respective units of force and area adopted.

The simplest possible state of stress is that of a short pillar compressed by load applied at its ends or the tension of the hanger of a bridge floor caused by its load. The first state of stress may be classified as simple longitudinal push or axial compression. The second as longitudinal pull or axial tension. The total pull (P) divided by the resisting cross section (A) equals the unit stress, $= P/A$.

4. Axial Forces. Let Fig. 1 represent a flat rubber band having a width *db*. Draw upon it a square *abcd* with the diagonal *ac* upon the axis of the band. Suppose the band is stretched lengthwise by the pull P. When the band is stretched in the direction of the pull it draws in laterally an amount *dd'* on one side and *bb'* on the other. It is stretched longitudinally the distance *cc'* so that the original square is elongated into a rhombus *ab'c'd'*. The material has undergone angular deformation *bab'*, *dad'*, similar to that under pure shear, as illustrated in Fig. 2, and shearing stresses are generated in the material along planes oblique to the axis of the direct stress.

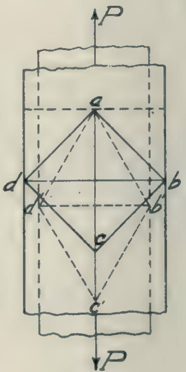


Fig. 1

The total axial force P which acts longitudinally across any right cross section $d'b'$ of the band acts in the same axial direction across any oblique section making an angle v with the axis. It exerts a total normal component on this section of $P_1 = P\sin v$, and a tangential component $P_2 = P\cos v$. If the area of the right cross section is A, then that of the oblique section is $A/\sin v$. Hence the intensity of normal component is $P \sin^2 v/A$ and of the tangential component is $s_s = P \sin v \cos v/A$. The maximum value of s_s occurs when $v = 45°$ in which case $s_s = \frac{1}{2}P/A$ is one half the intensity on the right section.

Any distributed force applied to a body can be resolved into normal and tangential components at the surface to which it is applied. When components produce normal compression they are positive. When they produce normal tension they are negative.

Tangential components which act to produce angular rotation in a clockwise direction are considered positive and those tending to produce anti-clockwise rotation are negative.

Suppose the equal faces of an elementary square prism *abcd* of an elastic body be acted upon by equal tangential forces *T* or *T′*, along each of the four sides in the direction of the arrows.

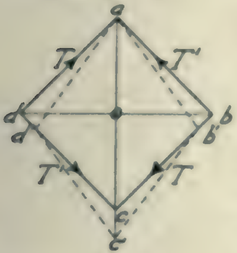


Fig. 2
Illustrating Deformation Produced by equal Tangential Forces on an element.

These forces will distort the square into a rhombus *ab′c′d′* and the total angular distortion of the area abcd will be twice the angle *bab′*.

As the original diagonal *ac* of the square prism has been lengthened to *ac′* the tangential stresses have produced tensions normal to *d′b′*. Because *db* has been shortened to *b′d′* compressions have been induced normal to *ac*. Consequently, tangential stresses produce not only angular deformation but direct tensile and compressive stress and strain as well.

5. **Moment.** The moment of any single force *T* (Fig. 2) about a point *O*, is the product of *T* by the shortest or perpendicular distance from *T* to *O*. This moment consequently depends both upon the magnitude of *T* and the position of *O* with respect to *T*. The distance of *O* from *T* is called the lever arm of *T* about *O*.

Couple. Two parallel forces $+T$ and $-T$, of equal magnitude but of opposite sign constitute a couple, and the algebraic sum of their moments is the same about any point whatever in their plane, and is called the moment of the couple. Its magnitude is the product of *T* by the perpendicular distance between the lines of action of the forces of the couple. This moment may be regarded as acting about any point whatever of the plane or any line normal to that plane.

Twist. Let any force *T* of a couple act parallel to the plane of the paper along the face *da* of an element *abcd* of a shaft which extends perpendicularly to the paper, and let the other opposing force *T* act along *bc*, then this couple furnishes a positive twisting moment tending to rotate the shaft in a clockwise direction. In case the rotation of the shaft is prevented the reaction is furnished by the internal resistance of the material, equivalent to a couple $\pm T'$ acting along the other two faces of this element *abcd* on some element parallel to *abcd* situated at some distance from *abcd* along the shaft. By such action twisting moments are transferred along the length of a shaft by means of the elastic resistance of the material. This elastic resistance consists of a twisting shear on all right sections of the shaft parallel to the face *abcd*. Thus twist produced by moment transfers shear and moment along an axis of moments, in this case along the length of the horizontal shaft transmitting power to any desired point, and the moment remains unchanged from point to point until a pulley is reached which reduces the moment by transmitting power from the shaft to a machine.

6. The Direction of the Axis of Moments may be defined as any line drawn perpendicular to the plane containing both the force and the lever arm. Consequently, **Moment is a Directed Quantity,** acting about an axis fixed in direction. Its amount may be laid off along that axis to represent it graphically just as a force may be laid off along its line of action. It can only be combined with the moments of other couples by combining their axes according to the parallelogram law in the same manner as forces are combined with each other.

For example, consider the case of equal positive bending moments acting about each of the axes of x and y that are each uniformly distributed in their action upon the respective edges of a unit elementary square piece of a thin plate thus tending to bend it about both of these axes at once. Then there is a total resultant bending moment acting on each diagonal which is found by multiplying that on one side by $\sqrt{2}$. But this moment is distributed along a diagonal whose length is $\sqrt{2}$ times one side. Hence the intensity of the moment per unit of length is the same along the diagonal as along a side, and it may similarly be proven that the intensity is the same on any vertical plane whatever.

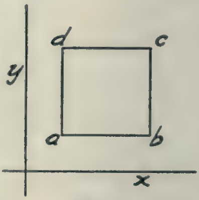


Fig. 3

When the equal moments about x and y are of opposite sign, the resultant moment would be of the same intensity on every plane but would not be normal to the surface on which it acts. It would then consist of a normal component which would be a bending moment, and a tangential component which would be a twisting moment. On planes at 45° with x and y the bending moment would vanish and the resultant moment would all be twisting.

7. External Moments. Moments of external loads and applied forces are designated as external moments. When a load upon a beam is multiplied by its distance from the support the moment thus obtained is that of the load about the support and by the principle of the lever when it is divided by the length of the beam the result is the magnitude of the reaction at the other support due to the load under consideration.

When the external bending moments are all about one and the same axis the resulting curvature is cylindrical and the bent surface is made up of parallel straight line elements. When the bending is in two directions and of opposite sign a saddle shaped surface is produced which is concave in one direction and convex in another at right angles to the first, and the applied moments involve twisting and warping as well as bending. The curvature of such a surface is called *anticlastic* as opposed to *synclastic* or dish shaped curvature such as a surface of revolution. Dish shaped curvature is produced by simultaneous bending in two directions when both curvatures are of the same sign.

8. Resisting Moment. The couple formed by the resisting stresses in the horizontal fibers of the material acting in tension and compression in a beam multiplied by a lever arm which is equal to the distance between the centroids of tension and compression respectively is the *internal bending moment* that opposes the external moment producing that bending. Likewise, where the external moment produces torsion or twist the internal moment opposing the external moment is the *internal twisting moment.*

9. Equilibrium of External Moments. I. The internal bending moment equals and balances the external moment of the applied forces where the member is bent in one direction only as occurs in cylindrical curvature, because there is no other kind of internal moment of resistance there present to produce equilibrium.

II. Where the moment of the external applied forces produces torsion or twist only as in a shaft, the external moment of the applied forces is balanced and equaled by the internal twisting moment of the shear couple since there are no other internal moments of resistance there present to produce equilibrium.

III. Where the member is bent in anticlastic curvature the external moments of the applied forces are balanced partly by the internal twisting moments and partly by internal bending moments because both are there present.

Deflections due to twisting or to shearing follow a fundamentally different law from those due to bending. For example, where twisting moments alone act as in a shaft, a simple summation of twisting deformations occurs along the axis of twist without increase in stress or twisting moment, which produces relative rotations whose amounts are equal to each other at the two ends of any portions of the shaft that have equal lengths. Likewise in a portion of a beam where bending moments of the same size act at all points there is a similar summation of shearing deformations horizontally along the length of the beam with equal increments of shearing deformation at equal distances from mid span, but the deflections follow an entirely different law.

10. Fluid or Hydraulic Stress: When an elementary cube is subjected to normal stress of the same kind and amount in three directions at right angles it is said to be subjected fluid or hydraulic stress because the intensity is the same in all directions.

Plane Hydraulic Stress, or fluid stress in a plane, are the terms applied to the condition of equal stress of the same kind in two directions within the material on planes normal to each other.

Pass any plane whatever through the line of intersection of the planes at right angles to each other on which there are equal normal stresses then the stress upon such a plane is normal thereto, and the intensity thereon is the same for every such plane as for the normal planes. For consider a rectangular prism whose length is one unit perpendicular to the paper, whose height and width are a and b respectively, subjected to a uniform normal stress of intensity S on a and b, then the resultant of the horizontal forces Sa and the vertical forces Sb will be $S\sqrt{a^2+b^2}=Sc$, acting on the diagonal c, in a direction perpendicular to c, and with an intensity $=S$ per unit of c.

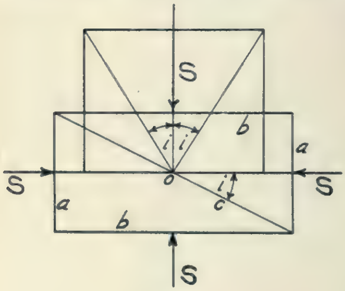


Fig. 4

The stress is evidently the same in all directions because we have assumed no fixed ratio between the length a and b and whether the prism be taken as wider or narrower the pressure on the surface c remains at S per unit of area, just as on a and b. Therefore, it is the same in all directions and is a true hydraulic stress in a plane.

Under plane hydraulic stress radial deformations and strains are equal in all directions, and circumferential stresses and strains likewise. In bending of plates this condition becomes a major element in accounting for their remarkable carrying capacity. The ultimate resistance of material under plane hydraulic stress is much greater than under normal stress in one direction only.

11. Right Shearing Stress. In Fig. 4, representing plane hydraulic stress due to equal normal stresses of the same kind on planes at right angles to each other, suppose that one of the equal stresses be made of opposite sign from the other. Then one of the components, say the horizontal component, is of opposite sign from that in hydraulic stress. The total resultant force, acting on the diagonal c remains unchanged in amount and intensity but no longer acts normal to c, but has normal and tangential components whose amounts depend upon the direction of c, Let c be inclined at an angle i to the horizontal then the resultant stress makes an angle $2i$ with the stress normal to c. When $a=b$ the cross section of the prism is square so that $i=45°$ and the plane of c makes an angle of $\pm 45°$ with the normal stress, the normal component in c vanishes, and the stress on both diagonals is entirely tangential. This is called a right shearing stress. (N. B. Every possible state of plane stress may be resolved into a hydraulic stress of a certain intensity superposed on a right shearing stress of suitable intensity). A state of right shearing stress at a point may therefore be defined either as consisting of a combination of two shears of equal and opposite intensity on planes at right angles to each other, or as a combination of a tension and a compression of this same intensity on each of two planes at right angles to each other making angles of $\pm 45°$ with the two shear planes first mentioned.

7. Equilibrium of an Element Under Shear. Because action and reaction are equal and opposite every tangential shearing force on the face of an element requires an equal and opposite shearing force on the opposite face. Two such shearing forces produce a rotary couple which must in some way be held in equilibrium or balanced by distributed or concentrated forces acting on opposite faces of the element or elsewhere. In Pure Shear the balancing forces are equal and opposite shears on the faces at right angles to the first pair of shears, and such distribution of stress may occur under flexure where a change of slope results from the rhomboidal deformation of the square elements and in pure torsion where such deformation occurs in the sides of the prism as before noted. But at those points along the length of a member under flexure where external restraint prevents rotary change in slope or horizontal shearing distortion, a different distribution of internal stresses is brought about by which equilibrium is maintained. For example, the supports of continuous beams built into columns are points where the tangent to the elastic curve is horizontal, and since rotary change in slope cannot occur, the balancing distribution of tensions and compressions at these points is brought about by external restraint. Such distribution would prevent rotation of *ad* about *d* Fig. 5 under the opposing shears *F* and *F*, shown in the figure. Uniformly varying stresses like those shown in Fig. 5 due to the rapidly varying bending moment at the supports will furnish the couple required to preserve equilibrium in this case. In fact any couple due to normal or tangential stresses whose moment is equal and opposite to that of the initial shears would maintain equilibrium.

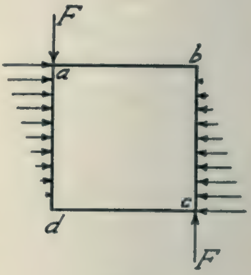
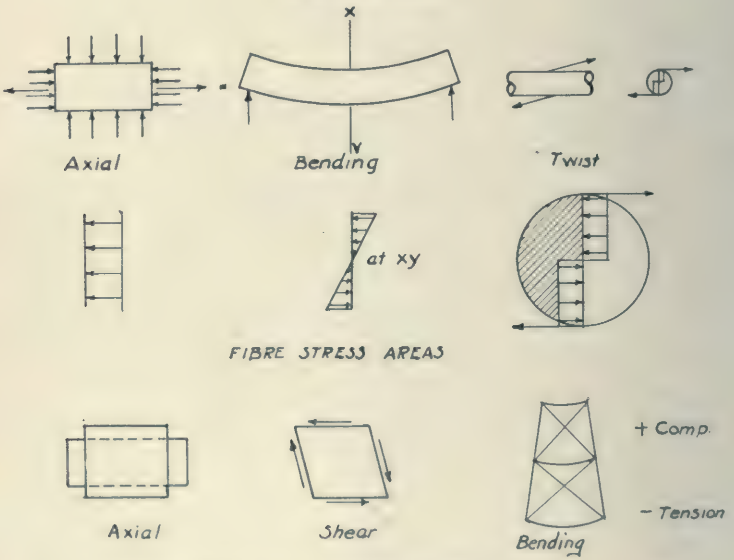


Fig. 5

Stress distribution normal to a plane section may be represented in intensity and direction by lines corresponding to the direction of the fiber stress of length proportionate to their relative intensity with the sign or direction of the stress indicated by arrows:

Fig. 6
ELASTIC STRESS



TYPICAL ELASTIC STRAINS

13. Fiber Stress Areas.—The relative areas covered by lines of length proportionate to stress intensity are termed *Fiber Stress*

Areas because they represent the relative intensity of the stress and its distribution over its plane of action. The Fiber stress area for axial stress is rectangular. For pure torsion symmetrical rectangles, for bending at the section of maximum moment, a triangle for the compressions and a triangle for the tensions. In torsion the symmetrical fiber stress areas about the center are of like sign. In bending they are of unlike sign, *e. g.*, compression on the concave side and tension on the convex side.

Characteristic deformations or distorsions of the square face of an elementary prism are—

(*a*) Deformation from square to rectangular takes place under axial stress in one direction or axial stresses of unequal intensity in two directions. The stress is uniform on opposite and adjacent faces though of different magnitude on adjacent faces.

(*b*) Trapezoidal deformation occurs in bending by uniform variation in intensity of normal stress on two opposite sides of an elementary square. In the compression zone of the beam at mid span each diagonal of the trapezoid is shortened while in the tension zone at mid span both diagonals are lengthened.

At the end of the simple beam the distortion is that of pure shear *i. e.*, the square face has become a rhombus in which the directions or axes of principal strain have shifted from the normal to the diagonal of the elementary square.

14. Moment of Inertia.—(Rectangular) Fiber stress area times breadth equals stress volume. Stress volume times lever arm about an axis in the section equals the resistance of the section about that axis. The resistance measured on a relative scale $f = \frac{1}{2}d$, equals the rectangular moment of inertia of the section. About XX, Fig. 7, the fiber stress area $= df/2$. Stress volume $= b \times df/2$. Lever × stress volume $= bdf/2 \times 2d/3 = bd^2f/3 =$ bending resistance of the sectional area about XX. As f increases as d for the same angle, the relative resistance of the cross section may be written $bd^3/3 = I$. This is virtually treating f as a unit represented on a scale of force equal to d as a unit on the scale of dimension of area. If XX is at the center of total depth d', and the stresses above and below are of opposite sign, then $d' = 2d$ of the preceding formula, and

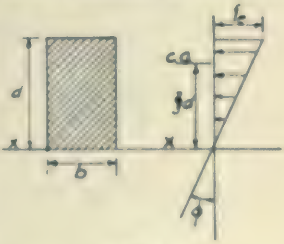


Fig 7

Stress resistance $=2b(\frac{d}{2}')^2f/3=bd'^2f/6$, which on a relative scald of $f=\frac{d'}{2}=bd'^3/3-bd'^3(\frac{1}{2}d')^2=bd'^3/12=I_0$

The least moment of inertia about XX is obviously that thru the center of gravity of the section.

The moment of inertia about any parallel axis from the derivation of I_0 is obviously—

$$I'=I_0+Ah^2$$

in which I' is the moment of inertia desired, I_0 is the moment of inertia of the area about an axis parallel to x thru its center of gravity. A = the area (bd) and h = distance from XX to the center of gravity.

The relative resistance of the cross section is thus the sum of the products each elementary area by the square of its distance from the axis of rotation and is stated in mechanics in the following form:

$I=\int_o^d\int_o^b y^2\,dy\,dx=b\int_o^d y^2dy=\frac{1}{3}bd^3$ as before found but its direct relation to bending resistance appears none too clearly understood by the average student.

Polar Moment of Inertia.—J Angle ϕ Fig. 8 is constant; hence unit shearing stress f is constant. Conceive the section divided into annular rings of a small fraction of the radius in thickness δr. The area of each ring wouldbe $\pi r\,\delta r$ and the total shear forces tangential to its face would be $f\pi rdr$. Since the area is made up of all such rings between $r=o$ and $r=R$.

Circular Section

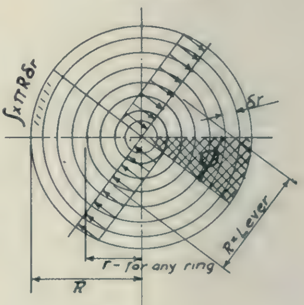


Fig. 8

$$\text{Stress Area}=\int_0^R \pi rdr=\frac{\pi R^2}{2}$$

$$\text{Stress Volume}=\frac{\pi R^2 f}{2}$$

Therefore, Stress Resistance $=\pi R^3f/2$, since the lever is R. Stress resistance on a relative scale may be written.

$$J=\frac{\pi R^4}{2}=\text{Polar Moment of Inertia.}$$

The polar moment of inertia of a square shaft is readily obtained from the fact that the rotational resistance must be constant thru 360°.

Stress volume on short diameter $= \frac{1}{\sqrt{2}} \times$ stress volume long diameter but the lever arm long diameter $= \frac{1}{\sqrt{2}}$ lever for short diameter. Hence $J = b^4/6$, quals the sum of the rectangular moments of inertia about XX and YY.

Moments of inertia for customary shapes and form are tabulated in all rolling mill handbooks which the student should secure.

15. Maximum Fiber Stresses, f.

(*a*) Pure bending equilibrium requires M, the External Moment $=$ Internal Resistance. $\therefore M = 2bd^2f/12$ or $f = 6M/bd^2 = \frac{M\frac{d}{2}}{I}$ Placing $d/2 = c$, the distance of the extreme fiber from the axis, $f = Mc/I$ the formula for the intensity of the extreme fiber stress in bending, which is to be kept within safe working limits for the material employed.

(*b*) Maximum Fiber Stress, Torsion: External twisting moment $M =$ Resisting Moment.

$$M = \frac{2\pi R^3 f}{4} = \frac{\pi R^3 f}{2}$$

$$f = \frac{2M}{\pi R^3}$$

*Note:—Erroneous reasoning regarding fiber stress in torsion is commonly found in college text books. Quoting Merriman's Mechanics of Materials, 11th ed. p. 227: "Shearing unit stresses (shown in the plane of section) vary as their distance from the axis of the shaft." We have shown unit shear to be constant, and present the following proof by Rankine.

Let R be the external radius. Conceive the cross section A to be divided into narrow concentric rings each of breadth dr. The area of the ring will then be $2\pi r dr$. The total shearing force $q = fr/R$.

$$q \div 2rdr = \frac{fr}{2\pi r dr R} = \frac{f}{2\pi R dr}$$

$\therefore$ the unit stress is a constant because the ratio of r/R cancels out.

$$M = 2\pi f \int_0^R r^2\,dr = \frac{\pi f R^3}{2} \text{ or } \frac{2M}{\pi R^3} = f^{*}$$

as before found in a simpler manner

---

[*] Altho the unit shear stress remains constant the spiral tension caused by it increases as the distance from the center of the shaft.

16. Circle and Ellipse of Stress.—Radial representation of stress intensity in a plane about a point furnishes a simple means of analysis and resolution of stress.

When the state of stress is the same in all directions (i. e. fluid stress) the radii of a circle described about the point at some convenient scale correctly represents the intensity of the stress in all directions about the point on the relative scale adopted.

Since stress intensity is the sum of the components, tangential (shear) or axial tension or compression along each radius the circle may represent either equal stresses at right angles of like sign or equal stresses at right angles of unlike sign, e. g., either a state of fluid stress or a state of right shearing stress.

The figure representing fluid stress is a circle, and an ellipse for the unequal stresses at right angles

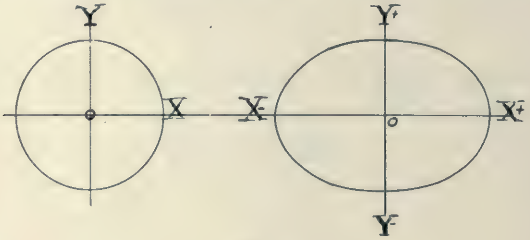


Fig. 9

Starting with the circle of stress in which $ox=oy$, suppose ox to gradually increase over oy. The circle gradually changes in form but the sum total of the tangents of inclination between o and x must remain the same.

The equation of the circle is

$x^2+y^2=R^2$, from which the tangent is

$$\mp\frac{dy}{dx}=\pm\frac{x}{y}$$

Since the material is uniform the curve must be uniform and at x and y the tangent must be o or oo, and hence the equation of the curve is $ax^2+by^2=a^2b^2$ i. e., an ellipse whose tangent

$$\mp\frac{dy}{dx}=\pm\frac{xa^2}{yb^2}$$

From such an ellipse the total intensity of the stress in all directions can be measured off. Its direction fixes its angle or obliquity to any plane so that stress components tangential and normal to any plane thru O may be at once determined by the parallelogram principle.

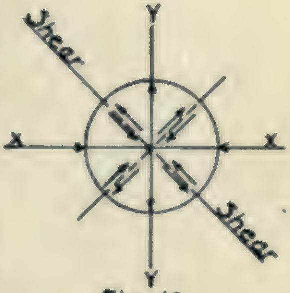


Fig 10

17. Pure Stress.—Consider the plane of action of stress thru the point to rotate thru 180°. In fluid stress all components are normal and none tangential. In a right shearing stress, at 45° to the greatest ± stress normal components are zero, and the shears change sign each 90° of rotation, and the stress at the point of change of sign is called pure because there is no oblique component as the shear there passes thru zero and the stress is pure (entirely) normal. Such a stress is called a *Principal Stress.*

Likewise at 45° the normal components pass thru zero and the stress is PURE (entirely) tangential, and these are the lines of greatest shear.

PROP: Principal stresses are orthogonal (at right angles) to each other. Let P_x and P_y represent the intensity of the numerically greatest and least stresses in a plane thru the point *o*. Construct an ellipse of stress. Then

$$P_x = \frac{P_x + P_y}{2} + \frac{P_x - P_y}{2}$$

$$P_y = \frac{P_x + P_y}{2} - \frac{P_x - P_y}{2}$$

and the state of stress is readily separated into a fluid stress $\frac{P_x + P_y}{2}$ in all directions and a right shearing stress $\frac{P_x - P_y}{2}$. As there is no shear on any plane in the fluid stress it is necessary to consider only the state of right shearing stress as was done in the preceding article, in which it was shown that the shear changes sign at planes at 90° to each other. Therefore, principal stresses, i. e., (*a*) a pair of pure tensions, or (*b*) a pair of pure compressions, or (*c*) a pair of pure stresses, one a tension and one a compression at right angles to each other define every state of stress in a plane about a point thru 360°. Q. E. D.

Cor. In a solid every state of stress about a point is defined by three orthogonal principal stresses, for if in a horizontal plane *xy* thru a point the two principal stresses *oy* and *ox* are known by the foregoing proof, *oz* must be orthogonal to *oy* and *ox* in the planes of *xz* and *yz*. Q. E. D.

18. Lines of Greatest Shear.—Where the principal stresses are of uniform intensity, the lines of greatest shear make an angle of 45° to the principal stresses or the principal stresses bisect the angle between the lines of greatest shear as in Fig. 10. When the principal stresses vary in intensity, the lines of greatest shear incline toward the principal stress which is decreasing most rapidly in intensity, but the principal stresses still bisect the angle between the lines of greatest shear.

19. Curves of Displacement and Equi Potential.—Curves of greatest displacement and least displacement called equi-potential are orthogonal lines which are determined jointly by the lines of greatest shear and principal stress in such wise that these curves bisect the angle between the line of greatest shear and principal stress.

Where the stresses are of varying intensity from point to point, the curves of displacement and equi potential are still orthogonal to each other because the principal stresses are orthogonal to each other and bisect the angle between the lines of greatest shear. Hence these curves which bisect the angle between the lines of greatest shear and principal stress must also be orthogonal to each other. The proposition is almost axiomatic since the resultant or greatest displacement is determined jointly by the greatest shear and the principal stress in its quadrant.

20. Combination and Separation.—Any states of stress which coexist at the same point and have their principal stresses in the same directions xx and yy combine to form a single state of stress whose principal stresses are the sums of the respective principal stresses lying in the same directions xx and yy; and conversely any state of stress can be separated into several coexistent stresses by separating each of its two principal stresses into the same number of parts in any manner, and then grouping these parts as pairs of principal stresses in any manner whatever.

The truth of this statement is necessarily involved in the fact that stresses are forces distributed over areas, and that as a state of stress is only the grouping together of two necessarily related stresses, they must then necessarily follow the same laws of the composition and resolution of forces.

For the sake of brevity, we shall use the following nomenclature of which the meaning will appear without further explanation:

The terms applied to forces and stresses are:-	The terms applied to states of stress are:-
Compound	Combine
Composition	Combination
Component	Component state
Resolve	Separate
Resolution	Separation
Resultant	Resultant state

(Eddy Researches Graphic Statics)

Problem.—Given any state of stress defined by two principal stresses px and py to find the direction of obliquity of the stress upon any plane thru O whose trace on xy is AB.

Let OX and OY be the directions of the principal stresses, Fig. II and Fig 12 OX being the direction of the greater stress.

Let Px be the intensity of the greater stress:

Let Py be the intensity of the lesser stress:

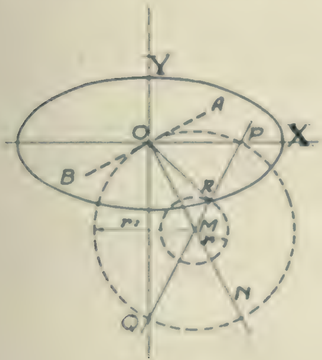
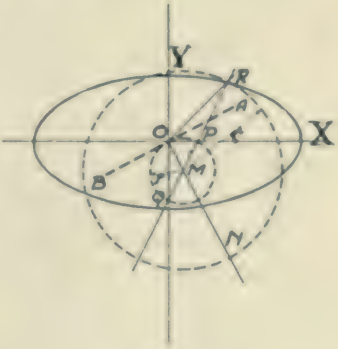


Fig 11



Fih. 12

Fig. II represents the case where the principal stresses are of like sign and Fig. 12 where they are of unlike sign.

Separate the state of stress into a fluid stress whose intensity $r_1 = \dfrac{Px+Py}{2} = OM$ laid off normal to AB. With radius OM draw a circle of stress intersecting OY at Q. Draw QM extended. With radius $r_2 = \dfrac{Px-Py}{2}$ describe a circle of right shearing stress of about M as a center and the intersection of QM with this circle will fix the point R. Connect RO; then will RO represent the intensity, direction and obliquity of the stress upon AB at the point O.

Thus the locus of the point M as the plane AB is revolved in a circle of the radius of $\frac{Px+Py}{2}$ represents the fluid stress into which the state of stress defined by the principal stresses Px and Py has been separated and MR the radius of the circle of right shearing stress as its conjugate component. The locus of the point R is the ellipse of stress otherwise derived from the law of variation of tangents of ellipse and circles of stress in Art. 15.

$$\text{The intensity of } OR = \sqrt{Px^2 . Cos^2 \widehat{XON} + P_y^2 sin^2 \widehat{XON}}.$$

The principal stresses being represented by the semi axes of the ellipse are the greatest and least of the plane stresses in XOY. The direct and shearing or normal and tangential components are found by letting fall a perpendicular from R upon OM as follows:

(*a*) Direct or normal $Pn = Pxcos^2\widehat{NOX} + Pysin^2\widehat{NOX}$.

(*b*) Shearing or tangential $Pt = (Px - Py)cos\widehat{NOX}sin\widehat{NOX}$.

From (*a*) the sum of the normal stresses on any two planes at right angles to each other normal to xy thru O is equal.

From (*b*) appears the equality of shearing stresses upon any pair of planes at right angles to each other, thru O normal to the plane XY.

(This relation was first demonstrated by Cauchy.)

21. The Principal of Conservation of Maximum and Minimum Fiber Stress Areas within the ellipse of stress appears at once from the discussion of Art. 19.

The plane of action of the lesser principal stress is the major axis and its stress area $= Py$ times $2OX$. The plane of action of the greater principal stress is $2OY$ and its area $= Px \times 2OY$.

Since $Px : Py :: OX : OY$ by construction, therefore the maximum stress area $Px \times 2OY = Py \times 2OX$ the minimum stress area. In like manner it may be shown that within the ellipse of stress the major and minor normal fiber stress areas in a plane thru O upon any orthogonal pair of planes normal to the plane of stress XOY are numerically equal

CHAPTER II

ACTION OF EXTERNAL FORCES AND REACTIONS OF ELASTIC RESISTANCE.

1. The applied forces distributed over the cross section of the resisting material perform the work of deformation against the elastic resistance of the material. By Newton's third law, this action and reaction are equal and opposite to each other.

Since mechanical work is the product of a force times the distance through which its point of application moves against resistance, the EXTERNAL WORK OF THE LOAD is its mean intensity multiplied by the deflection of the elastic structure or deformation of the material in the direction of its application.

Conversely, the INTERNAL WORK OF DEFORMATION of a unit of material is the product of the mean stress times the strain deformation produced by the stress and the total work of deformation is the sum total of the work of deforming the several parts or all the units of the specimen.

2. Energy (Mech.) is a term used to define capacity for performing work, distinguished as kinetic or *actual energy* in reference to the work which in a given case it actually performs; and as potential energy in reference to the work which it is capable of performing if fully exerted.

Actual energy is exemplified in the *vis viva* (active energy) of moving bodies in heat, electric currents, etc. Potential energy is exemplified in the resistance of a spring bent downward by a gradually applied load ready to raise the mean load as it is gradually removed the same amount that during loading the position of the mean load was lowered. It requires mechanical work to pump water to any height and when so pumped up it has energy of position capable of doing work as it runs down to a lower level as through a turbine wheel.

In the case of a spring or beam bent downward under a gradually applied load, the decrease of potential energy of position of the load by loss of height is stored up in the elastic resistance of the spring or beam ready to be entirely restored in raising the load during gradual removal if the spring or beam be perfectly elastic. But if it be imperfectly elastic, then some part of the energy of position lost by the load cannot be restored to the load during gradual unloading because it has been transformed into heat in overcoming non-elastic resistance within the spring or beam. It is then lost with respect to the system embodying the load and spring only but not destroyed since the heat has been radiated to the surrounding air.

In the case of the water tower, actual energy expended in raising the water by pumping is stored as potential energy by increased head of the water in its elevated position.

A transfer of energy from one part of an elastic system to the other occurs under change of load but the total gain or loss in one part is exactly equivalent to the loss or gain of the remainder in accord with the fundamental law that energy may be changed in form but cannot be destroyed.

Applied to a bridge it expresses the relation that the loss of energy of position of the loads by deflection is transformed by the elastic resistance of the structure into internal work of deformation.

3. **Analysis.**—The principle explained in Art. 2, is of great value in the analysis of stress and computation of deflection of structural members under applied loads. The method of analysis based directly thereon is termed *The Theory of Work.*

ANALYSIS BY THE GEOMETRY OF ELASTIC DEFORMATION is founded upon proportionality of cause and effect. When the cause of deformation is stated in units of mechanical work then elastic opposing resistance may be expressed in the same units, and is equal thereto if no work has been done in permanently changing the structure of the substance.

This may be shown experimentally by loading a spring balance on which each division of the scale represents one ounce with a quantity of shot. By placing sixteen grains (weight one ounce) on the pan, the pointer indicates a certain downward motion of the spring registered as one division or ounce on the scale. Placing sixteen more on the pan, a further downward motion of another space occurs, and two divisions are registered, and so on until with 256 shot on the pan it registers sixteen divisions or ounces. If now, the shot be removed grain by grain, on the removal of 16 shot, the spring has moved 240 grains of shot up one space and the dial indicates 15 ounces. If we remove 16 more, the spring moves 224 grains upward one space more and registers fourteen divisions or ounces, and so on until they are all removed.

The mechanical work of the mean weight times its downward motion during gradual loading of the spring equals the mechanical work, i. e., product of the mean load times the distance through which it is raised by the spring during the step by step process of unloading. Therefore, the external work of the load equals the elastic internal work of deformation stored in the spring as shown by its performance in raising the load as step by step removal took place.

Since the static work of elastic deformation equals the sum of all mean stress times the corresponding strain equals the gradually applied mean load times the downward motion in loading and proportional relations exist between the load and deflection, stress intensity and magnitude of strain must be likewise proportional.

The foregoing elastic relation is known as *Hooke's Law* first published by Robert Hooke, in 1678,—Ut tensio sic vis, as the extension, so the resistance, or freely translated, "Extension is proportional to force (applied)."

4. Elastic Limit.—The reversible cycle of the experiment of the preceding article shows that the elastic limit is that range of gradually applied loading within which the elastic deformation stores up the entire external work of the applied load in such manner that it can restore this energy in the step by step process of unloading. When it is exceeded the deformation cannot be wholly expressed in units of the same kind of energy because a part of the external work has been expended in over coming non-elastic resistance and has been converted to and dissipated in heat units which cannot follow the reversible cycle.

Elastic return to the original form or shape after strain is a characteristic deportment when stress is kept within the elastic limit. But it in no wise constitutes a determinant of that limit since return to the original form may be unaccompanied by the capacity to restore the work of deformation. A constant ratio of the magnitude of deformation to the intensity of the applied load or force producing the deformation is a true determinant so that the true elastic limit is also a *Limit of Proportionality*, as well.

The limits of elasticity for steel and materials of construction are so narrow that the distance between any two neighboring points of the substance never alters more than a very small proportion of its own amount without either breaking or experiencing a permanent set or inelastic deformation.

For ordinary steel the elastic elongation or shortening before permanent set takes place varies from 1/800th to 1/1000th of the length and the angular distortion not far from two tenths of a degree.

It accordingly appears that the determination of the elastic limit experimentally by the determinant of proportionality gives approximate values because of the small magnitude of the elastic strains involved and the difficulty in making sufficiently precise measurements.

5. Potential Energy of Deformation.—An elastic body in a given state of strain is capable of doing a certain amount of work against external resistance as in the experiment in unloading the spring balance. Under the principles of statics of abstract forces this work of raising the load during the process of gradual removal would constitute the potential energy of the state of strain which the spring had undergone during the process of loading. Elastic reactions to applied forces, however, exhibit strain components normal to the applied forces and since no static component exists in the normal direction the principles of statics of external forces do not quantatively fix the potential energy of deformation as equal to the energy expended by the applied force or load. Since the latter energy comprises the sole external source the body itself must have suffered a transfer of kinetic internal energy to make up this apparent increase of internal potential energy of deformation by lateral deformation.

Now the kinetic internal molecular energy of any substance is measured by its temperature and its relation to stress and strain will now be investigated.

6. The Mathematical Theory of the Thermo-Elastic Properties of Matter has been developed by Kelvin, (Quarterly Mathematical Journal, April 1855) and the following formula which is applicable to a metal bar is there demonstrated:

$$H=\frac{t\,p\,e}{J\,S\,W};$$

where H = thermal change in degrees Centigrade;
t = temperature, Centigrade, from absolute zero;
J = the mechanical equivalent of the thermal unit, = 1400 ft. lbs. per degree Centigrade;
p = pressure applied, in pounds, positive if compression and negative if tension;
e = the longitudinal expansion per degree Centigrade;
S = the specific heat;
and W = the weight, in pounds per foot of length of bar,

A digest of Kelvin's work in Thermo Dynamic Theory is given in Todhunter & Pearson's History of Strength and Elasticity, Vol. II, Chapter 14.

This change in temperature, as would be inferred from the mechanical theory of heat, takes place instantaneously, with the application of the pressure, or load, p, and is subject to the usual laws of radiation. Since the temperature change is very small, Newton's Law of Cooling may be taken to apply exactly, namely; The amount of heat gained or lost by a body in a given time is proportional to the difference in temperature between the body and the surrounding medium. Hence if two loads, of different magnitudes, are gradually applied in equal periods of time, the temperature changes in the bar for the respective loads will be strictly in proportion to the loads, notwithstanding the radiation going on while the loads are being applied.

The formula

$H = \frac{t\,p\,e}{J\,S\,W}$ is rigidly exact for any solid only for an infinitesimal increment in p; but with steel, any variation from it within elastic limits would be extremely small, and may be taken as exactly proportional to the pressure or load applied, and, as we are interested only in relative measurements, no error is involved.

As H depends on t, the temperature of the bar at the time the load is applied, this temperature should be noted, if readings taken at different temperatures are to be compared. Thus, if the value of absolute zero be taken at —276.7° Cent., and one set of measurements is taken at 23.3° Cent., and another at 24.3° Cent., the latter, for the same loads, should be 1/3 of 1% greater, if no error has been made in the determination. As t would probably not vary greatly, this percentage per degree Centigrade is sufficiently accurate for most purposes.

The magnitude of the temperature change, H, may be best illustrated by a few figures. Take the temperature of a bar at 19.3° Cent., p at 1 lb., and the area of the bar at 1 sq. in., and we have, for mild steel,

$H = \frac{(276.7+19.3)\times 1.0\times .00001142}{1400\times 0.116\times 3.34} = 0.00000623$ Cent., the temperature change for a change in stress of 1 lb. per square inch.

Such minute temperature changes as those resulting from the application of a load of even one quarter ounce per square inch may be measured by thermo electric means and it is therefore possible to determine the range of true elasticity of such a homogeneous material as steel by this method with a high degree of precision. Such measurements and the apparatus employed have been described by the author, (Trans. Am. Soc. C. E., August 1902). The apparatus used was an antimony-bismuth thermopile, insulated from and applied to the polished surface of the specimen, in circuit with a D'Arsonval type of galvanometer with the usual scale and reading telescope fordetermination of the relative intensity of the current produced by different loads.

Suitable insulation of the thermo-pile from the animal heat of the observer, from radiant heat of electric light, and disturbing air currents is essential as well as the insulation of the pile to prevent any short circuit from contact with the polished surface of the bar. Material for the latter insulation was the subject of much study, rubber cement proving the most available material since it gave an exceedingly thin coat which did not interfere with the transmission of heat and effectually prevented short circuit.

It was found necessary to vibrate the scale beam of the testing machine in order that the weighing of the load should correspond with the degree of accuracy of the electrical measurements. From these experiments Turner shows that the limit of thermal proportionality in mild and medium steel is approximately five eighths of the yield point value of the metal. The bar was found to grow gradually cooler under increased loading until this limit was reached when the heat generated by internal molecular friction off-set the increment of reduced temperature and the thermal curve deviated from a straight line more and more until the yield point value was reached when the sudden evolution of heat at that point would make the reading fall beyond the scale.

So in compression tests. The readings were found to be proportional up to approximately a similar limit when the increase in temperature of the bar would no longer follow the right line changes but would increase more and more rapidly as the yield point was approached but at that point the reading would be too great to fall within the limits of the scale.

By the foregoing formula we may proceed to compute approximately the relation of the heat transformed from kinetic energy of molecular vibration to potential energy of elestic strain and compare its amount with the work of the mean external force or pull times the stretch of a bar of steel.

Take the section 1″ square; E, the ratio of unit pull divided by unit stretch, at 30,000,000; applied pull 10,000 lbs. per square inch.

In a foot, Elongation $e = \frac{4}{1000}$

External Energy = Q_x = .002 × 10,000 = 20 in. lbs.

Internal Energy = 00000623 × 10,000 × 1400 × .116 × 3.34 = 33.3 in. lbs. = 34.7 lb. ins. = Q_1.

As the transfer of kinetic internal molecular energy to the potential energy of strain or vice versa is much greater than the applied energy of the external forces it immediately appears that temperature change under stress is an important factor or partial determinant in fixing the final change in form or shape of an elastic body under stress.

As the temperature is lowered by pull below that of the surrounding air by radiation therefrom (the stress remaining constant meanwhile) the temperature is brought back to its original amount before strain, e. g., the elastic material has received energy from without. Conversely if the strain be compression by radiation more energy has been dissipated than expended in compressing the bar, to be returned to it again by radiation from the surrounding air in the reversible cycle of loading and unloading.

Since the ultimate ratio of static stress and strain is in part a thermal function, the relations of stress and strain present two different aspects or ratios.—

(*1*) A geometric ratio (called kinetic by Kelvin in Elasticity, Ency. Brit.,) which is the instantaneous ratio of elastic stress divided by the strain, and

(*2*) A static or thermal ratio which represents the final static after the strained specimen through radiation from or to the surrounding unchanged medium has arrived at the temperature thereof.

These ratios will be termed respectively the GEOMETRIC (i. e., kinetic) and STATIC MODULII OF ELASTICITY.

The so-called lag of strain in reaching the final or static state follows Newton's law of radiation.

The rapid decay of elastic vibration is predominantly caused not by viscosity or friction with the external air but by the necessary transferance in elastic change of form of nearly double the quantity of kinetic energy of molecular vibration in heat units to potential energy of strain.

CHAPTER III.

THE MECHANICS OF ELASTICITY.

1. A body is said to be HOMOGENEOUS when the material is of the same kind or nature throughout. A strain is said to be HOMOGENEOUS when parallel lines thru the same particles before strain remain parallel after strain.

Uniform intensity of both principal stresses in a plane cause *homogeneous states of strain.* Varying intensity of both principal stresses cause heterogeneous states of strain in which parallel lines in the body before strain are no longer parallel after strain.

A body is said to be ISOTROPIC when it presents the same elastic properties in all directions. Such a body rigidly obeys Hooke's law, and opposes equal resistance to stress in all directions.

Fibrous materials and metals which have been heavily rolled or otherwise over strained differently in different directions are unequally strong in these respective directions and are said to be EOLOTROPIC.

The investigation will deal first with the properties of the ideal isotropic material and then follow as far as possible the cause of Eolotropy and the resulting modification in resistance and deportment of material incident thereto.

2. **The Kinetic or Geometric Modulus** of the stress strain relation is to be determined from the geometric relations of the lines of principal stress and greatest shear to the curves of resultant displacement and equi potential. Conversely, the relative geometric modulus may be derived by separation of the state of strain in a plane into the four states of displacement of which any given state of strain is the resultant. See Art. 18 and 19, Chapter I.

The simplest case of homogeneous strain will now be considered: Let a flat bar, Fig. 13, be subject to an axial pull P:

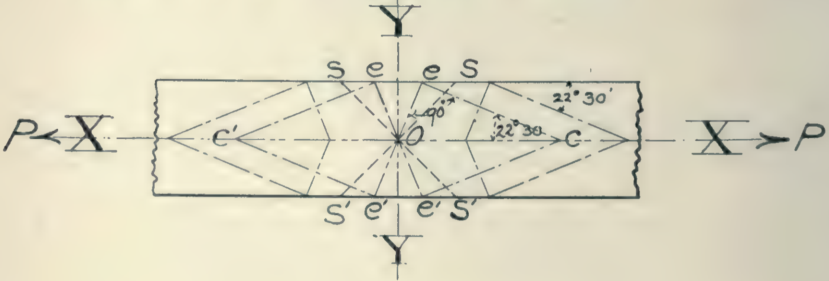


Fig. 13

The greatest elongation is from *O* (the center) toward x_1 to the left and from *O* toward x_2 on the right. The planes of greatest shear are *OS*, *OS′*, at 45° to *OX*. The major principal stress is parallel to *OX*, the minor principal stress is parallel to *OY*. The curve of *equi potential* bisects the angle between *OS* and *OY* on each side of *O*, i. e., the angle $eOY = 22°30' = e'OY$.

The curves of displacement are *e′c′*, *ec*, etc., at 90° to *eo* and *e′o* respectively. The angle which *e′c′* and *ec* make with the boundary *ee′* is 22°30′.

In the quadrant *xOY* positive there is accordingly a ellipse of displacement whose major axis is inclined to OX_2 at an angle of 22°30′ and whose minor axis is inclined to *OY* at an angle of 22°30′.

The ellipse of displacement in quadrant *XOY* negative has its axes rotated thru 45°.

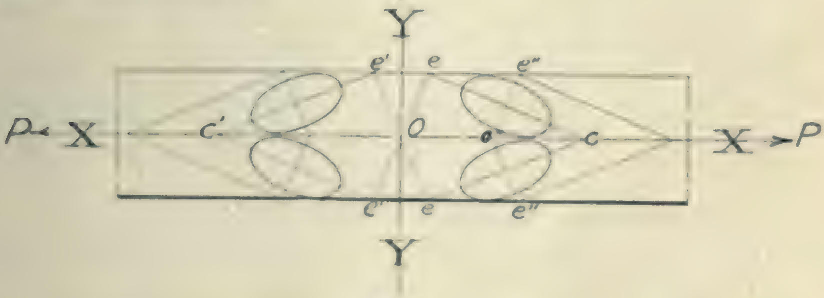


Fig. 14

Ellipses of displacement in the four quadrants of an axial state of stress are shown in Fig. 14.

Compounding the ellipses of displacement in the four quadrants the resultant state of strain presents its major axis coincident with *OX* and its minor coincident with *OY*.

The properties of the different ellipses of displacement are comparable to the properties of the ellipse of stress Art. 20, Chapter I, with respect to the conservation of resultant displacement and equipotential areas. Therefore, the material of the bar has suffered a total kinetic compression that is exactly equal to its total kinetic elongation.

Therefore, VOLUME IS A KINETIC OR GEOMETRIC INVARIANT OF ELASTIC STRESS.

In the plane of *XY* the stresses along *OX* are balanced by the reactions of the resultant elongations and compressions of the ellipses of displacement of adjacent quadrants to the right and left of *OY* in the plane of *YOX*. The same state of apparent balance appears statically in the normal plane of ZOX and accordingly this double potential of strain must draw its energy from the intrinsic energy of molecular vibration or heat of the body in the original state of zero stress. The potential energy of the strain in the isotropic uniform solid is momentarily twice as great as the quantity of energy expended in producing the strain. But the intrinsic heat energy of the unstrained body has suffered a loss equal to twice the external work of deformation.

3. Young's Modulus E and Poisson's Ratio K.—(Kinetic) While in the ellipse of displacement of the isotropic elastic body strain is strictly proportional to stress, the axes of this ellipse are not parallel to the boundary of the bar and it is desirable to know the elongation in the direction of the pull rather than along the curve of resultant displacement; also the contraction normal to the boundary plane rather than along the curves of equi-potential. The elongation in the direction of the pull is figured as unit pull over the coefficient E called Young's Modulus. The isotropic relation of E to the coefficient of lateral contraction may be determined from the principle that volume is a kinetic invarient of the elastic strain as follows:

Let dx be a small increment of elongation;
Let dy be the corresponding increment of lateral contraction;
Then for kinetic invarience

$$(1+dx)(1-dy)^2=1$$
$$(1+dx)(1-2dy+dy^2)=1.$$

Evaluating $dy/dx=.4142=tan 22°30'$ equals the angle of inclination of the curve of displacement to the boundary plane, equals the ratio of unit lateral contraction divided by unit elongation in the direction of the pull $P=$ the coefficient K or Poisson's ratio.

Problem: (Let the student determine the same relation from the geometry of stress curves and the curves of displacement.)

4. Rotational or Shear Resistance is measured by a coefficient called the *Modulus of Rigidity F*. As the radius of rotation is the lever of the force causing rotational displacement or strain the relative value of F depends on the relative length assigned the lever. In the case of the bar, Fig. 13, the lateral contraction occurs from the edges toward the center and the shear distortion is half above and half below the axis. The squeezing together of the material viewed from this standpoint is twice as easy as though the rotational distortion were all in one direction. Thus under axial pull there is no applied shear force, hence half P produces a rotation or action in one direction and half P produces its reaction in the other direction. Under transverse stress the actions and reactions are divided between external rotation and internal shear strain. As compared with the unit upon which the elongation is conveniently measured the lever of the shearing force is 1/*cos* 45, consequently the isotropic relation of $E/F = 1/2 \times 1.414 = .35355+$. This value of F is the only value consistent with the conception of isotropy as opposing equal resistance tò stress in all directions.

THE MODULUS OF RIGIDITY is commonly defined (for isotropic or eolotropic material) as the ratio of the intensity of a right shearing stress to the amount of the shearing strain produced by that stress. It is denoted by the letter F. Where the shearing strain is small it is equal to the tangent of the angle of deformation and for a shearing stress of intensity q pounds per square inch we then have

$$\text{shear stress} = \text{shear strain} \times F, \text{ or}$$

$$q = \text{tang } \phi \times F = \phi \text{ (in radians)} \times F$$

$$F \text{ (pounds per square inch)} = \frac{q}{\phi} = \frac{\text{shear stress in lbs. per sq. in.}}{\text{shear strain in radians}}$$

Let *abcd* be the square face of a unit cube of material which is firmly fixed at *c* and *d*, and has a horizontal tangential or shearing stress of intensity q applied to its upper surface *ab*. This will bring into play an equal and opposite shearing resistance along *cd*, as well as equal and opposite vertical reactions at *c* and *d* which are equivalent to shearing stresses of intensity q on the faces *bc* and *da*. These all together deform the square *abcd* into the rhombus *ab′c′d′* Fig. 21 in which the angular deformation

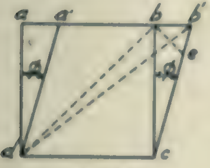


Fig. 15

$$\phi = \frac{aa'}{da} = \frac{b'b}{bc} = \frac{q}{F} \text{ as above.}$$

The elongation of the diagonal *db′* is *eb′* cos 45° and the linear strain along each diagonal is

$$\frac{eb' \cos 45^\circ}{db} = \frac{\frac{1}{2}bb' \sqrt{2}}{cb \sqrt{2}} = \frac{bb'}{2cb} = \tfrac{1}{2}\phi = \frac{q}{2F}$$

But this linear strain along the diagonal *db* is one half the shear strain just found for the side *ab*. This right shearing stress is eqiuvalent to an axial tension along *db* and an equal axial compression along *ac*. These together may be regarded as producing the angular deformation ϕ of which the tension produces one half as shown in Fig. 16 and similarly the compression produces the other half. Morever, the tension may be regarded as producing one half of the intensities of all of the shears as in Fig. 16 and the compression the other half. Fig. 15 may be made comparable with Fig. 16 by rotating $a'b'c'd$ about d thru the angle $c'dc = \frac{1}{2}\phi$ as shown in Fig. 16.

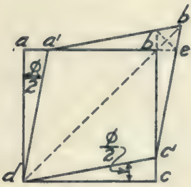


Fig. 16

12. Effect of Like and Unlike Stresses, Acting at Right Angles. The ultimate resistance of material under plane hydraulic stress is greater than under direct stress of the same magnitude acting in one direction only. Hydraulic stress is brought about by stress of equal intensity of the same kind acting on planes at right angles to each other. Each stress produces an axial or normal strain in the direction of its application and a lateral strain of opposite kind normal thereto. Consequently, two normal stresses acting on planes at right angles to each other produce strains, which, since they tend to off-set each other do actually each diminish the other in the ratio of the axial strain to the lateral strain characteristic of the material under consideration.

But on the contrary, where two stresses of opposite sign act on planes normal (at 90 degrees) to each other, their coaction serves to increase the strains which either acting separately would produce. See Art. 2 and 4.

Because of this increase of the true strains over those computed to occur when normal strains are disregarded, **safety** requires the precise investigation of the amount of the increase or decrease of these stresses by lateral deformation in case of stresses of opposite sign on normal planes. Further, precise translation of deformation into true stress is requisite if elastic theory is to be checked by the measured elastic deflection of the structure under load, or if computed applied stress is to be compared with extensometer measurement of true strain. Without such precise translation, analysis by the elastic theory remains unsubstantiated, and its accuracy or inaccuracy is a matter of mere opinion rather than a matter of fact to be determined beyond dispute by scientific test.

13. On The Determination of True Stresses From The Measured Strains by Help of Elastic Coefficients

The modulus of elasticity E as just defined is the longitudinal unit stress p acting along x say, divided by the corresponding longitudinal unit strain e, in the same direction, viz. $E=p/e$, on the express condition that there is no lateral applied stress perpendicular thereto along either y or z. Hence longitudinal stresses can be correctly calculated as the product of the observed longitudinal strain e and the modulus E only in case there are no lateral forces acting along either y or z.

In this case of simple longitudinal stress there is, however, a lateral deformation or unit strain amounting to $-Ke$ along both y and z, and the apparent or applied stress p acting along x produces not only the longitudinal strain e along x but also two lateral strains each equal to $-Ke$ along y and z respectively.

In order to find out how much longitudinal strain the stress p would produce in its own direction x were it not also employed in producing lateral strain along y, let us superpose on the body a lateral stress of intensity $+Kp$ along y while the stress along z remains unchanged at zero. This applied stress of $+Kp$ along y when acting alone produces a lateral strain $+Ke$ along y and $-K^2e$ along z, but when acting in conjunction with the existing lateral strain $-Ke$ makes the resultant deformation along y vanish, while the total strain along z will be due to the stress p along z and to Kp along y, and hence will amount to $-K(1+K)p$ along z.

This deformation $+Ke$ along y which has been superposed on the body is necessarily accompanied by a superposed strain of $-K^2e$ along both x and z.

Hence the resultant strain along x due to the longitudinal stress p when lateral strain along y is entirely prevented and no stress is applied along z, amounts to $(1-K^2)e$, and the true stress along x as indicated by extensometer measurements of the strain will be $p_1=(1-K^2)p=(1-K^2)Ee$, which is the intensity of that part of the total applied stress p as calculated by help of E which produces strain in its own direction x. The rest of the applied stress along x, viz. $+K^2p$ is held in equilibrium by resistance to lateral flow represented by the lateral stress $-Kp$.

The foregoing calculation of true stress along x applies to all cases where the intensity of the lateral stress actually applied along y is such as to prevent deformation along y, while that along z is negligible, under which conditions the true stress tending to deform the body in the direction of x does not exceed $(1—K^2)p$ and the strain in that direction does not exceed $(1—K^2)p/E$.

Again, take the case of an applied longitudinal stress of intensity p along x and two applied lateral stresses along y and z of intensity q respectively. The resultant deformations will be those due to superposition of the effects of the separate stresses, viz.

Stress	Strain Along		
	x	y	z
p along x	p/E	$—Kp/E$	$—Kp/e$
q along y	$—Kq/E$	q/E	$—Kq/E$
q along z	$—Kq/E$	$—Kq/E$	q/E
Total strain	$(p—2Kq)E$	$(q–K(p+q))/E$	$(q–K(p+q))/E$
True stress	$p—2Kq$	$q—K(p+q)$	$q—K(p+q)$

The foregoing may be applied to the case of hooped columns, whenever it may have been determined either what is the absolute magnitude of the hydraulic stress q in the plane y z, or what fraction q the lateral stress may be of the longitudinal stress p.

The foregoing may also be applied to the case where q is of opposite sign from p by changing the sign of K, in which case the stresses and strains are increased instead of being diminished as they are when p and q have like signs.

14. Relations Between Elastic Constants. The foregoing investigations give rise to certain relations between E, the modulus of direct longitudinal strain, F the modulus of shearing strain, and K the coefficient for lateral deformation under direct stress. As already shown, the strain along the diagonal of a cubical block of material subjected to a state of right shearing stress of intensity q on two planes at right angles is $\frac{1}{2}\frac{q}{F}$, which may be replaced by $\frac{1}{2}\frac{p}{F}$ where p is the intensity of equal and opposite stresses normal to the diagonal planes, because the intensity is the same on all planes passing thru the line of intersection of the shear planes as already shown.

The resulting direct stress p, in the direction of one diagonal only would, if acting alone, cause a strain $\frac{p}{E}$ in the direction of that diagonal, while the opposite kind of direct stress in the direction of the other diagonal perpendicular to the first, would, acting alone, cause a similar kind of strain to the above, amounting to $K \times \frac{p}{E}$ in the direction of the first mentioned diagonal. But each of these direct strains would be accompanied by a lateral strain of the opposite sign amounting to Kp/E. It is to be noticed that the two strains $\pm Kp/E$ at right angles to the plane of the right shearing stress annul each other. Hence the total strain along either diagonal is

$$\tfrac{1}{2}\frac{p}{F} = \frac{p}{E}(1+K)$$

from which $\dfrac{1}{2F} = \dfrac{1}{E}(1+K)$

$$1+K = \tfrac{1}{2}\frac{E}{F}$$

For steel as received from the mill—
$E = 3(10)^7$ $K = .3$ and $F = 115(10)^5 = 3846E$.

When relieved from internal stress by repetition of stress from tension to compression of low intensity K approaches .4142 and F .35355E while E increases toward $315(10)^5$ from $3(10)^7$.)See Bauschinger's experiments).

8. Kinetic and Static Modulii Compared.—To find the ratio of the kinetic or geometric to the thermal or static modulus the dimunition of strain due to the temperature change H, equals eH. Hence if E' denotes the static modulus the strain when the body is not allowed to gain or lose heat is $p/E' - eH$, or with H replaced by its value

$$\frac{p}{E'} - e\frac{tpe}{JWS}$$

Dividing p by this expression, the kinetic modulus =

$$E' = \frac{1}{\dfrac{1}{E'} - \dfrac{te^2}{JWS}}$$

In Art. Elasticity, Enc. Brit., Kelvin gives for compression

$E = 1.00259$ for iron
E' 1.00325 for copper

These differences, though small, are of sufficient magnitude to be considered in any precise experimental investigation.

9. Summary: From the foregoing development, the true value of K for a constant volume has been geometrically determined and the relation of curves of displacement to the lines of principal stress and greatest shear developed for the first time.

The value of potential of elastic strain was not understood by the founders of the mathematical theory of elasticity, Navier, Poisson, and Cauchy, and for that reason they supposed the true value of K to be .25 instead of .4142. They derived these equations from Boscovich's hypothesis of intermolecular action. A review of their work correcting it in the light of the double transfer of Kinetic energy of vibration to strain potential would have led them to the results here presented and avoided the fruitless controversy between the rari and multi constant schools of elasticians as Pearson has designated them in his great treatise.

Proof has been adduced for the first time that an isotropic solid can have but one independent coefficient of elasticity.

CHAPTER IV.

PHYSICAL PROPERTIES OF MATERIALS OF CONSTRUCTION, THEIR FITNESS FOR DIFFERENT KINDS OF SERVICE, WORKABILITY AND SAFETY.

1. Properties.—A material is said to be ductile when it can be drawn out or elongated a considerable percentage of its length before final fracture. It is said to be brittle when it breaks without such plastic elongation or snaps short under a blow or shock. As the hardness of the material increases as a rule its ductility decreases. Thus, soft steel will stretch twenty five or thirty percent of its length before fracture, while hard tool steel shows little plastic deformation before rupture. Cast iron for example snaps in tension without noticeable plastic elongation. Building stone, which is relatively hard, strong in compression and shear, is brittle and unreliable in tension.

That material is preferred for resisting axial tension or bending in which the ultimate resistance in tension and compression is equal. Thus, mild steel, which has equal resistance in tension and compression, is most suitable for resisting axial tension or bending. Brick, stone and artificial stone, such as concrete, which present reliable compressive resistance, are used in masonry. Cast iron, which in the past century was used extensively for lintels, and columns in buildings, is being replaced by structural steel though still most extensively employed for machinery, cylinders, gears, pipe, fittings, grate bars, stoves and furnace castings.

The following brief outline of the properties of materials will deal more particularly with the characteristics which are related to elasticity and which cause variation in their strength, toughness and utility.

2. Brick is made of burned clay. The clay is prepared by mixing with water, moulding into the desired form by hand or machine and after the brick are dry they are placed in the kiln and burned until those nearest the fuel assume a partially vitrified appearance. The quality of the brick taken from the kiln are first, arch brick, those formed around the arches forming the furnace chamber of the kiln, body brick from the interior, soft brick from the exterior. Arch brick are hard and often brittle; body brick are the best quality, while the soft brick are suitable only for filling in or backing up where a high degree of strength is unessential. The common size is $2\frac{1}{2}$x4x$8\frac{1}{4}$ inches. The average weight about $4\frac{1}{2}$ pounds. Pressed brick may weigh nearly $5\frac{1}{2}$ pounds. Soft brick will commonly absorb twenty to thirty percent of its weight of water while body brick about half that amount.

Fire clay makes the strongest and most impervious brick. Since the strength of soft brick runs as low as five to six hundred pounds, and the stronger brick up to fifteen thousand pounds per square inch, statement of average resistance of brick work is of little utility and a corresponding variation in the modulus of elasticity E occurs with the corresponding difference in the material. In general it may be said that the strength of a brick pier laid up on cement mortar or portland cement mortar is about one third that of the average individual brick. Brick is never used to resist tensile stress. Few experiments have been performed to determine its resistance under such stress. With the softer varieties it is probably not much greater than eighty to ninety pounds running up to a thousand pounds or more for better varieties of fire clay brick.

3. Building Stone.—Building stone is divided into three classes, classes, sedimentary, metamorphic and igneous.

Sandstone, as its name implies, is usually a sand the grains of which have been chemically cemented together under pressure and heat. Where the chemical action and heat have been sufficient the sandstone is changed into a quartzite; it varies greatly in strength, color and durability. In general, it is easy to cut and dress but where it has been transformed into pure quartzite or that variety known as Potsdam sandstone it may be hard and correspondingly difficult to cut. The softer varieties, when first quarried may be worked with much greater ease after they have been exposed to the air. In seasoning they grow much harder and stronger. Limestone is frequently formed from marine shells and varies greatly in strength and durability. Some oolitic limestone containes layers of clay or hardened sea bottom slime which causes gradual disintegration when exposed to the elements.

Marble is a limestone which has undergone a high temperature and subsequent crystalization. In composition it is nearly pure carbonate of lime. It is easily worked, can be highly polished and makes desirable building material.

Granite is an igneous rock composed of quartz, feldspar, and mica, but in that variety called syenite the mica is replaced by hornblend. It is strong and durable, provided there is no iron in its makeup. It takes a high polish. Many quarries of syenite have been so subject to pressure that cleavage planes extend thru the material at convenient intervals for quarrying blocks of desirable size for building purposes.

Trap, or basalt, is an igneous rock without cleavage. It is hard and tough, and less suitable for building construction than the preceding varieties of stone because of the difficulty in cutting large blocks to size. The material is of high strength and possesses remarkable durability and weathering qualities. Merriman gives the following table of the average weight, compressive strength and shearing strength of these four varieties and assigns a modulus of elasticity ranging from 5,000,000 to 10,000,000 pounds per square inch:

Kind	Pounds per cubic foot	Pounds per square inch	
	Weight	Compressive Strength	Shearing Strength
Sandstone	150	5,000	1,200
Limestone	160	7,000	1,500
Granite	165	12,000	2,000
Trap	175	15,000	2,500

The quality of building stone cannot be safely inferred from mere tests of strength. Its weathering qualities depend on its capacity to resist the action of frost which may be better determined by investigating the material where it has been exposed for a long period to the weather or wherever the stone has been under conditions of actual use. The strength of a stone pier laid up in good Portland cement mortar may be taken about one fourth to one fifth that of the stone itself.

4. Concrete.—Concrete is an artificial stone formed by a mixture of Portland cement, sand and crushed stone or gravel as aggregate. The strength varies with the richness of the mixture, grade of the sand and strength of the coarse aggregate. A coarse sand with gradation in sizes of the grains gives the strongest concrete, and the stronger the aggregate, up to the strength of the mortar, increases the strength of the whole. Common mixtures are in the proportion of one cement, three sand and five stone for walls, footings and piers, while a 1:2:4 mixture is common in concrete which is strengthened or reinforced by steel rods to supply the tensile resistance which the concrete lacks. When so reinforced, the composite or heterogeneous material may imitate to some extent the action of the homogeneous material of equal tensile and compressive resistance. The departure from the stress distribution of homogeneous material as the section passes from the steel to the concrete, possessing entirely different characteristics, is to be investigated subsequent to the analysis of homogeneous material under transverse stress, torsion, and combined stresses.

The utility of material such as plain concrete, stone and brick, in which there is such a wide divergence between the tensile resistance and the compressive resistance, resides in the fact that the shearing resistance is far greater than the tensile resistance generally. The principles governing the relations of the tensile and compressive resistance to shear resistance will now be considered.

5. Modulus of Rupture and Ultimate Shearing Resistance in Materials in Which the Compressive Resistance Greatly Exceeds the Tensile Resistance.—The conception of the resistance of isotropic material, presented in Chapter III, Art. 1, and the discussion of the modulus of rigidity F, Art. 4, might lead to the conclusion, could the internal stresses be considered like abstract forces acting upon an elementary prism as tending to approach a value between two and three times as great as the tensile resistance if ultimate failure could be attributed to mere lack of tensile strength. But as tension and compression approach equal values under the conception of the isotropic resistance, the material should present the same resistance to shear that it does to either tension or compression. Again, since shearing distortion is made up jointly of tensile and compressive resistance, and even when over strained in tension, the tensile stress volume must still equal that in compression, a shift in the position of the neutral plane necessarily occurs in order to make up this balance of resistance.

The value of the elastic fiber stress which would furnish the same resistance at failure is called the modulus of rupture in transverse bending and is equivalent to the resistance of pure shear at rupture. this value appears to be a geometric mean between the value of the ultimate resistance in tension and that in compression. Thus cast iron, having a tensile resistance of 20,000 pounds per square inch; and compression resistance of 80,000 pounds per square inch, the modulus of rupture should be about 40,000 pounds on a one inch square bar under concentrated load. This relation is the geometric modulus for a state of ease—i.e., it is modified by a state of shrinkage strain.

In the concrete beams tested by the Bureau of Standards, in many samples the loads were concentrated at the quarter or third points.

With such distribution the modulus of rupture would be less than the geometric mean because while the moment is constant from mid span to the load, horizontal shear deformation increases from zero at mid span to the load, tending to reduce the resistance and induce cracking of the material under and in the vicinity of the load.

A 1 : 3 : 5 concrete, for example, having an ultimate compressive strength of 2,500 pounds per square inch and a tensile strength of 300 pounds, should have an ultimate shearing resistance as a geometric mean of 866 pounds per square inch. A 1 :2 : 4 concrete, having ultimate strength of 3,500 pounds per square inch, and a tensile strength of 350 pounds, should possess on this basis, an ultimate shearing resistance of 1,107 pounds.

6. Timber.—From the standpoint of the elastician, timber is a fibrous material, presenting different resistances to stress dependent on the direction of the grain. Its crushing strength transversely and longitudinally of the grain are different as is also its shearing resistance.

While the dividing line is not sharp, timber is classed as soft or hard wood. Common varieties of the soft woods are the cone bearing trees such as White Pine, Norway Pine, Hemlock, Spruce, Douglas Fir, Long and Short Leaf Yellow Pine, Cedar and Cypress. The common kinds of hard wood are White, Red and Black Oaks, Live Oak, Hickory, White Ash, Maple, Gum, Locust, Elm, Black Walnut, Lignum Vitae, Iron Wood, Mahogany, Teak.

Hard woods when cut for timber are stripped of the bark which unless removed hastens decay. Inside the bark there is a soft portion made up of thin wall cells which constitute the living or growing portion of the tree, called the sap wood. The inner portion is harder and stiffer than the outer sap wood. The growth of each season consists of fibers added to the circle of the sap wood. This growth is more rapid and open in the spring and slower and more compact during the summer, giving rise to peculiar ring marking of the fibers which indicate each year's increase in growth. The rings inside of the sap wood form the heart wood. They are interrupted by radial cells connecting between the center of the tree and soft tissue of the outside. These radial cells are called the medullary rays. In the Pine, the sap wood consists of forty to sixty percent of the cross section of the tree, and the time required for sap wood to transform into heart wood ranges from a few years with the fir, to many years with the oak.

SAWING: The manner in which the lumber is sawed from the log has a marked influence on its quality and behavior. The method of sawing is determined by the character of the wood and the purpose for which it is to be used.

Flat sawing consists in roughing the log tangentially to the annular rings and finishing with parallel kerfs to the side of the log. Rift sawing is cutting the boards in such manner that the annular rings are cut through as nearly as possible in a radial direction.

Rift sawing and flat sawing give rise to the term edge grain and flat grain. Edge grain lumber shrinks and checks less, and wears more evenly and smoothly without slivering than the flat grain lumber. It is sometimes termed quarter sawed.

NATURAL SEASONING: In the preparation of lumber for construction purposes, it is necessary to dry it out or season it to prevent decay and excess shrinking. Natural seasoning consists in exposing the planks or boards after sawing to a free circulation of air by piling across three or four narrow strips placed in the opposite direction to which the layers of plank are placed in parallel. The time required for thorough seasoning varies from one to two years, dependent on the character of the wood.

ARTIFICIAL SEASONING or kiln drying hastens the evaporation of the moisture but tends to impair to some extent the strength and elasticity of the wood. The timber is stacked in a kiln and exposed to a current of hot air, the temprature dependent on the kind and dimensions of the stock. Sometimes the heat is supplemented by the employment of vacuum pumps. The temperature used varies from 100° F., for oak, to 200° F., for Pine.

SHRINKAGE: The average values for lateral shrinkage in wood are for soft pine, cedar, cypress, etc., three percent; for hard pine, ash, oak, walnut, maple, locust, or white pine, four to five percent. Birch, beach, locust, hickory, young oak, five to six percent. Longitudinal shrinkage is usually less than one per cent.

Timber swells and shrinks with its moisture content. Wood block pavement has been known to expand five to six inches in swelling of the blocks in a forty foot width.

VARIATION OF STRENGTH WITH MOISTURE: Effect of thoroughly soaking the timber on its compressive strength is to reduce it twenty percent or more. In other words, timber is strongest in tension and compression when it is thoroughly dried out. The best timber is secured by cutting in the fall and winter, when the sap is not running.

ELASTICITY AND TOUGHNESS: White oak, white ash, hickory are exceedingly tough and springy, when the timber is cut in its prime and for archery, bows equal to the famous English long bow may be formed from this timber if cut in its prime. On the other hand, where the trees are old and past their prime, the timber loses its toughness and elasticity to a large extent and becomes brash, or brittle. Failure of wooden automobile wheels in general is due to brash timber and the failure to properly inspect the stock before working it up. The following table gives the properties of common varieties of timber.

*PROPERTIES OF TIMBER.

Kind of Timber	Wt. per cu. ft. dry Lbs.	Strength in lbs. per sq. in.				Shearing lbs. per sq. in.	
		Tension	Compression		Bending Modulus of Rupture		
			With Grain	Across Grain		With the Grain	Across the Grain
Oak, Red and Black	43	10,000	6,000	2300	9,100 15,400	1100	......
Oak, White	46	15,300	8,000	2000	6,000	750 1000	2300
Oak, Live	62	8,200	9,000	 2700	 5,400	 1000	8480 6000
Hickory	49	15,800	8,500	3200	24,300 6,300	1200 450	7800 6280
White Ash	47	14,000	6,500	1900	14,200	1100	
Maple	45	9,000	8,000	1700 1900	....	......	6350
Gum	37	16,500	8,000	1400	6,000 12,700	8000	5890
Locust	44	17,600	9,300	....	11,200	.....	7176
Black Walnut	38	12,500	7,500	..	....	.....	4828
Lignum Vitae	41 to 83	11,000	8,800		12,000	....	.....
Iron Wood	60	15,000	12,000	....	12,000 19,000	...	..
Whitewood	30	7,000	4,800		.....		4418
Mahogany	35	13,000	7,500	..	10,800		....
Teak	41 to 61	15,000	12,000		12,000 19,000		..
Pine	30 to 43	9,000	4,800	700 1400	5,000 14,000	300 700	4000 5000
Fir	32	12,500	7,300	800 1200	6,500 12,000	500 600	...

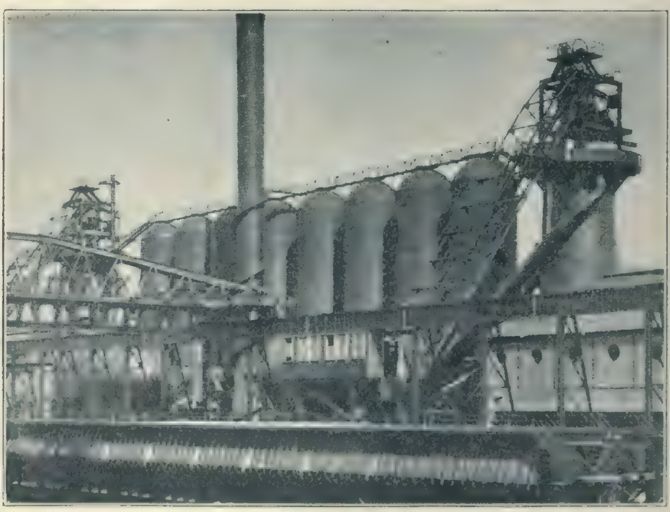
*The above table is compiled from various standard sources which do not disclose data regarding the conditions of the tests. The higher values were unquestionably obtained from small samples of heart wood in prime condition the lower values from tests of larger specimens which had not been cut in prime condition. Conservative, safe values for inspected timber are given on pages 83 and 84.

7. Iron and Steel.—If a lump of red or brown hematite be heated for a few hours in a charcoal fire well surrounded by or embedded in the fuel it will become sufficiently reduced to permit forging at a red heat into a bar of iron, which constitutes the primitive method of producing wrought iron from the ore, a method still practiced to some extent in India and Africa. It may be noted that by this method the first ore discovered in the Lake Supreior basin was reduced to merchantable iron.

While great economies have been effected in the consumption of ore and fuel by present day methods, no improvement has been made in the quality of wrought iron and high carbon steel. The old blades of Damascus, Toledo and Bilboa have never been excelled.

CAST IRON: was first produced in England during the fifteenth century, using coke as fuel and limestone as a flux in the blast furnace.

The molten metal is drawn off and cast in * sand moulds on the furnace floor into blocks of convenient size called pigs. Pig iron high in carbon is called foundry pig and is gray in color, more readily melted for casting into desired form in foundry practice. Pig relatively low in carbon is termed forge pig and is utilized in the manufacture of wrought iron and steel.



Modern Blast Furnace

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*Casting machines have taken the place of sand moulds where the moulten pig is not charged into a converter or open hearth furnace.

8. Wrought Iron.—Following the introduction of cast iron wrought iron was more economically produced by the reduction of forge pig in the puddling furnace. The hearth of a reverbratory furnace is covered with a layer of high grade ore or mill scale which acts as an oxidizing agent for reducing impurities. The reduction occurs at different stages. During the melting down stage much of the silicon and manganese and some of the phosphorous are removed. During the boiling stage under stirring or puddling most of the carbon and the remaining phosphorous and sulphur are largely removed. As the mass becomes purified it assumes a pasty condition. The metal is then rolled into balls and carried to the squeezer and there most of the slag is expelled.

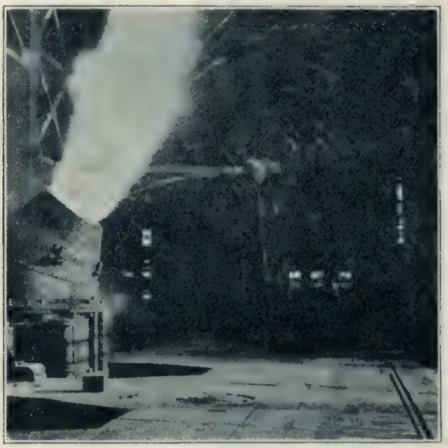
The bars from the first rolling are known as muck bars. These are cut into lengths, piled, heated to a welding heat and rerolled, and the product is merchant bar iron. If again cut, piled, and rerolled the product is called "best" iron. A third rolling gives "best-best" iron, superior in quality but higher in price.

Prior to 1890, wrought iron was generally employed for structural purposes but medium steel, on account of its smaller cost and greater uniformity and strength, then came into use so rapidly that it has today almost entirely displaced wrought iron in bridge and building construction. Wrought iron is still used for machine parts where its extra cost is more than compensated for by its weldability and the superior toughness of best-best iron under severe service.

Its properties were so thoroughly investigated during the period of its extensive use in the 19th century that it will long remain in engineering literature a standard with which other materials may be compared.

The structure of wrought iron is fibrous in a matrix of ferrous silicate welded together with an intervening cinder of magnetic oxide. The percentage of the cinder content varies from one to one and one half percent. The silicate resists corrosion while the magnetic oxide if unduly thick in the interior or as surface scale promotes corrosion. The iron fibers separated by the cinders afford a combined resistance to shock superior to soft steel of the same ultimate strength. The tensile strength of good wrought iron runs from 46,000 to 50,000 pounds per square inch, its yield point about 24,000; coefficient of elasticity 25,000,000.

9. Steel.—The displacement of wrought iron came about thru the development, subsequent to 1870, of the Bessemer or pneumatic process of conversion. The pig is melted in a cupola furnace and the molten charge poured into a cylindrical refactory lined container called a converter, mounted on trunions and provided with a number of tuyeres in the bottom for the introduction of the blast. Feromanganese is then added to furnish the desired carbon content of the steel. High grade ore is necessary for the Bessemer process while the rapidity of conversion renders the control of the charge less certain than in the slower open hearth process.



The Bessemer Converter Flame at its Height.

Though the process as first introduced in England by Sir Henry Bessemer in 1856 produced practically pure iron in a fluid state, the metal poured from it was full of blowholes and largely unfit for use. In Sweden the process worked well from the start and investigation led to the final step necessary for complete success. The Swedish iron was found to contain as much as five percent of manganese and it was proved by adding manganese to the wild blown metal of the English mills that sound setting steel would be produced. As finally carried out, the practice was to add molten Spiegelisen containing ten to twenty percent of manganese and five percent of carbon. Thus, at one stage was added to the carbonless steel the desired percentage of carbon with the manganese.

Steel is said to be Acid or Basic, dependent upon the character of the furnace or converter lining. If the lining is silicious (acid) the steel is said to be acid. If dolomitic or of grain magnesite the steel is said to be basic.

OPEN HEARTH STEEL: In 1861, Charles, William, and Frederick Siemens originated the Open Hearth process, the growth of which has been phenomenal. In 1900, the reproduction of Bessemer steel was double that of the Open Hearth process; in 1908 Open Hearth tonnage exceeded that of Bessemer, while at the present time the Open Hearth tonnage is approximately three times that of the Bessemer process.

This rapid development was due to the growing scarcity of ores from which pig can be made either sufficiently low in phosphorus for acid Bessemer or sufficiently high for basic Bessemer. As none of the phosphorus is eliminated in the acid process, steel makers are limited to iron which contains only that amount allowable in finished steel. In the basic Bessemer this element must be sufficiently high so that its oxidation together with manganese and carbon can generate heat sufficient to keep the charge hot. Otherwise, what is known as a cold heat will result. The Open Hearth on the other hand is adapted to either basic or acid practice, utilizing pig iron and scrap of a wide range of analysis and is in no way dependent for its heat upon the temperature generated by the exidation of the impurities in their elimination from the molten metal.

As compared with the Bessemer process, the operations of the Open Hearth are under greater control and samples can be taken at intervals for both physical and chemical tests. There is less danger of over oxidation and as a whole the Open Hearth product is more uniform and reliable than that of the Bessemer converter.

With the exception of the bottom lining, basic and acid Open Hearth furnaces are identical. The lining of the hearth of the basic furnace is generally made of maganesite and the acid furnace of silica clay brick covered with a thin layer of sand which is glazed by turning the gas heat into the furnace. The operation is repeated until a bottom of the desired depth is obtained.

CONTINUOUS PROCESS: (Patented by Talbot in 1899). An Open Hearth furnace of large capacity is used often holding 200 tons of metal. The initial charge of scrap and pig is worked in the usual manner. When ready to tap, about one fourth of the metal is withdrawn. To the remainder is added oxide of iron in the form of iron ore or mill scale. After this has become melted, hot metal and limestone equal in amount to that already tapped are added. The slag, now rich in oxide of iron causes violent reactions and purification is quickly effected. The additions are made at intervals, dependent on the violence of the reactions. The proceedure is repeated as often as the metal is sufficiently purified. The method permits a considerable variation in the analysis of the metal and produces a large tonnage.

DUPLEX PROCESS: The Duplex process is a combination of the Bessemer and Open Hearth: It is particularly applicable in the use of pig iron containing too high a percentage of silicon for advantageous reduction in either the basic Bessemer or basic Open Hearth process. The preliminary treatment in the acid Bessemer container removes the silicon, considerable manganese and a certain amount of carbon. The molten metal then is transferred to the basic Open Hearth where the phosphorus is eliminated and the carbon reduced to the percentage desired. This process is employed mainly in the South where the iron produced is more adapted to this treatment.

SULPHUR: In rivets, eyebars, boiler and firebox steel, the presence of sulphur is objectionable, because when the metal is heated it tends to create a coarse crystallization thereby reducing the strength and toughness and rendering it brittle. For this reason the limiting maximum is commonly fixed at .04% to .05% for such use. Metals for forging or heat treating require similar limitation.

In ordinary structural material the effect of this element is of little importance, for up to as high a limit as .10% it appears to have no appreciable effect upon the elastic ratio, elongation or reduction of area.

PHOSPHORUS is objectionable in the rolling mill, for it tends to produce coarse crystallization and lowers the temperature to which it is safe to heat the steel and for this reason phosphoric steel should be rolled at a lower temperature than pure steel in order to prevent the formation of a coarse crystalline structure.

Although it increases the static load carrying capacity about the same amount as carbon up to .12 percent, it diminishes the ductility of steel under a gradually applied load as measured by its contraction and elongation when ruptured in an ordinary testing machine. Moreover, it diminishes its toughness under shock to a still greater degree and this it is that unfits phosphoric steel for most purposes, requiring rigid limitation of its content, as follows:

Structural steel for buildings	Bessemer	.10 Per cent
Structural steel for buildings	Open Hearth	.06 Per cent
Structural steel for bridges	Acid	.06 Per cent
Structural steel for bridges	Basic	.04 Per cent
Rivet steel		.04 Per cent
Flange steel	Acid	.05 Per cent
Flange Steel	Basic	.04 Per cent
Fire box		.04 Per cent

10. Physical Properties of Mild Steel.—The hardness and tensile strength depends largely upon the percentage of carbon. A rough rule for this strength is

$$T = 45{,}000 + 100{,}000C \text{ for acid steel}$$

$$T = 45{,}000 + 90{,}000 \text{ for basic steel}$$

T being the carrying strength in pounds per square inch, and C being the percentage of carbon.

Campbell, in 1905, deduced from exhaustive investigation the formulas:

$$T = 40{,}000 + 68{,}000C + 100{,}000P + 80{,}000CM \text{ for acid steel}$$

$$T = 38{,}000 + 65{,}000C + 100{,}000P + 9{,}000M + 40{,}000CM \text{ for basic steel.}$$

C being the percentage of carbon, P that of phosphorus and M of manganese.

Classified according to the percentage of carbon, steel is said to be soft when the carbon is under .20. It will harden little by quenching and is easily welded. Structural steel from .20 to .30C may be hardened somewhat and is weldable. Intergrade steel .30C to .40C can be hardened and welded. Hard steel .40 to .70C may be tempered, welded only with difficulty; .70 to 1.00C very hard, high temper not weldable.

DUCTILITY: For mild steel, up to .35C, the elongation in eight inches should be equal to 1,500,000 divided by the tensile strength. For hard grade steel, this value of reasonable elongation reduces to approximately 1,200,000 divided by the tensile strength.

The fracture of good steel should present a bright metallic luster, should be silky or finely crystalline. The nick and bend test gives a much better indication of the character of structure of the sample than the fracture under the tensile text. It instantly discloses the undesirable character of the product of the faggot mill with its coarse texture, dull color and streaks of steel extending through the iron base. This material, sometimes called BUSHEL STEEL is made by forming a faggot box with muck bars for the base and sides, filled with miscellaneous steel scrap heating it up and rolling into billets and bars. It will bend more readily than soft steel but is lacking in strength, elasticity and general fitness for structural purposes.

BEND TEST: Good soft steel of $\frac{3}{4}''$ bars and under should bend on itself 180 degrees without sign of fracture on the outside of the bend. For material over $\frac{3}{4}''$, around a pin whose diameter equals the thickness of the specimen. For over $1\frac{3}{4}''$ thickness, around a diameter equal to twice the thickness of the bar.

The percentage of elongation may be reduced about one percent for each additional 1/8″ in thickness above one inch.

11. Rerolling.—Heating, rolling and working a billet down in size improves the character of the steel as rerolling improves wrought iron provided the heat is uniform and not sufficiently intense to burn or injure the metal. The lack of uniformity in heating and rolling a billet commonly results from irregularity of form of that billet. Thus, in rerolling old rails, the metal may be intrinsically high grade but with an uneven heat the rail is frequently on the point of melting on the outside while heated to merely a cherry red on the inside. Splitting it and rolling it in this condition produces bars which are brittle and highly unsatisfactory. This fact is the basis of the prejudice against rerolled stock. It is possible by a slow heat to produce steel of fair quality from stock of this character.

12. Crucible Steel.—By far the greater portion of rolled and cast steel in use today is produced by the Bessemer and Open Hearth process briefly outlined in the foregoing discussion.

Special steel of high quality, used for cutlery, instruments and small tools which require high temper, special toughness or fine edge are made from special stock in a more expensive manner. Iron of a high degree of purity is first produced and then subjected to a process of recarburization. In the early days pure charcoal iron was hardened for high class tools by the cementation or as it might be called, the complete case hardened process, carried to the point where the carbon case penetrates the center of the bars. Pure iron bars were packed in charcoal in a cast iron box, then heated to a red heat in the furnace. The bars were kept hot from seven to eleven days and then allowed to cool slowly. In the raw state such bars are known as blister bar or cemented steel. Heated and hammered to longer bars they formed the spring steel of the earlier days. Blister bar is not uniformly carburized, the outside having more carbon than the center.

About 1740, Huntsman, a watchmaker of Sheffield, England, remelted blister bar in crucibles, producing a uniform product and made the first Crucible steel of England. In the East, crucible steel has been produced for centuries by methods similar to the modern American methods. Iron made directly from ore in a crude charcoal hearth or furnace is remelted in small clay pots with charcoal, the iron absorbing the carbon of the charcoal and in that way being transformed into high carbon steel. The more expensive method of melting cemented steel is now giving place entirely to the crucible product.

Bessemer and Open Hearth steel remelted in the crucible pot are superior to the same steel not so melted, but are inferior to remelted wrought iron. A part of the excellence of crucible steel is accordingly due to the method of melting and part to the quality of the raw material. That traceable to the method of melting arises from the fact that the steel is protected from the oxidising gases of the furnace by the pot and its cover so that the steel does not oxidise and absorb its own oxide. Nor does it absorb harmful gases as the contained air is soon exhausted by the oxidising charcoal or the carbon, silicon and manganese of the iron. The atmosphere in the pot thus becomes neutral, perhaps even reducing. The steel after melting absorbs silicon from the clay of the pot, the carbon of the steel reducing the silica to silicon and this absorbed silicon deoxidises the steel and quiets it. This action is facilitated or accelerated by the addition of ferro silicon and ferro manganese to the steel shortly before pouring.

The superior excellence of wrought iron as a base for high grade crucible steel may be accounted for by the reduced tendency to absorb oxides and gases at the lower temperature at which it is produced. The cinder contained in the wrought iron easily floats out of the steel in melting in the crucible. Although the purifying action of the crucible process improves Bessemer and Open Hearth steel, it is not as effective as the slow solidification in the puddling furnace and puddled iron further improved by the remelting remains superior to Bessemer and Open Hearth steel cleaned only by pot melting.

Cruibles are made either of clay, or of graphite and clay in about equal parts. Once heated, the clay pot must be recharged hot and the first heating must be very slow or the pot will crack.

Crucible steel making, unlike Open Hearth and Bessemer process, is a pure melting rather than a converting process. Within certain limits the steel maker is confined to so proportioning the material that the total percentage of their different elements, allowing for the gain or loss in melting, will be what is desired in the finished steel.

The process in principle is to melt practically carbonless wrought iron with enough carbon in the form of charcoal and other metal to give the steel the carbon content and alloy content desired. In the case of tool steel this will be from .50 to 1.8 percent carbon. The wrought iron can be figured to contain .10 carbon. No iron is suitable for high grade steel that runs over .012 per cent of phosphorous and sulphur, as only by using pure iron of this low content can steel be advantageously produced with these two harmful elements below .025 to .03 percent which the maximum limit for high grade steel.

Alloys, such as Tungsten, Molyddenum, Chrome, Nickel, etc., should be figured a little below their anaylsis content of metal to allow for oxidising losses in melting. Silicon and manganese in the alloys scrap and iron generally drop somewhat in melting.

Hall states that it is best to weigh the small amounts of the ferro silicon and ferro manganese required, using the highest percentage alloy that can be obtained at a reasonable price, to place the amount weighted out in small Manilla envelopes and have the melter toss them into the pots five or ten minutes before pouring. Used in this manner the loss of silicon and maganese is slight and by charging them at this stage the dead melting period can be greatly shortened as silicon is added in the required amount instead of being slowly absorbed from the clay of the pots.

13. Electric Steel Making.—Commercial application of electric heat to steel making is a very recent development, although pioneer electric furnaces were built about twenty five years ago by Siemens, and Stassano, later by Snyder, Herault and others. Only the heat

Four ton Booth—Hall Electric Furnace

energy of electricity is used in steel making. No electrolytic action is required. Accordingly, either direct or alternating current may be employed. As three phase alternating current costs less to transmit than single phase or direct current it is usual to transmit the power in three phase form and if desired to change the three phase to single phase or direct at the point where the current is to be utilized. To use three phase current directly in the steel furnace requires three electrodes.

The furnace charge is similar to that in the Open Hearth process, consisting of pig, ore and scrap. The pig supplies carbon. Cast iron scrap or carbon in the form of charcoal can be substituted for the pig iron. The ore furnishes oxygen. Mill scale or a blast of air can be used as an alternate supply of oxygen. Mill scrap, the cheapest material from which steel can be made is always added to the charge. This is especially true because the electric furnace can readily melt borings and turnings which work badly in the Open Hearth and Bessemer furnaces.

The electric furnace atmosphere can be made non-oxidizing so that such granular material is not burned and lost in the slag. Ore is the more expensive material for this kind of steel making. Not only does it cost more per pound of contained iron but it requires three times as much electric energy to make steel from ore as from scrap.

The materials charged carry with them various forms of impurities, the linings of the furnace melt in pots and drop into the charge. This with the constituents of the metal, such as silicon, and maganese combined with sulphur, oxygen, and phosphorus, form insoluable compounds in the form of slag. To bring the slag to the proper mechanical consistency fluxes are added, either silica or lime in powder or granular form. The amount is relatively small, the total slag generally weighing less than twenty percent of the weight of the metal.

Mechanically, an electric furnace is operated the same as an Open Hearth furnace. As the electric furnace works faster the furnaces are smaller in holding capacity for a given tonnage output.

Chemically, an electric furnace is the same as an Open Hearth, except the oxidizing influences of the gas flame are absent. The passage of current through the bath sets up movements in the metal which serve to mix it and render it more uniform.

Higher temperature can be obtained at reasonable cost with the electric furnace than with the fuel steel furnace. To these high temperatures is attributed the higher quality of the metal. The hotter slag separates more completely from the metal and the desired reactions are carried further toward completion in the time available for the heat. The result is a higher tensile strength for the same anaylsis and a higher value for the elastic limit and yield point of the metal.

When first introduced, electric steel was made at a high cost. Gradually the furnaces have become more efficient and simpler. Even, at the first, the cost of electric steel was said to be somewhat lower than Crucible steel for the same tonnage output. Recently such furnaces have been built in sizes that compare in output with the larger commercial sized Open Hearth furnaces. The low operating cost of the electric furnace together with the high quality of the product, renders probable a change toward electric steel making in the coming decade similar to that which took place in the last decade from the Bessemer converter to the Open Hearth. Where good ore and ample water power is available, at low cost, we may look for steel centers in the future where the scarcity of coal today has prevented commercial development of the steel industry by older methods.

14. Gray Iron Castings: Iron castings may be soft enough to be machined with greatest ease or chilled as hard as glass. In physical structure they may show a fracture from silver white to gray black. This wide variation is characteristics depends on the amount and condition of the carbon content. Roughly this will run from 2½ to 4 percent and may be either in a combined form as in the white glass hard iron for the one extreme or almost entirely uncombined in the form of graphite as a dead gray iron at the other extreme. Between these two limits there may be any combination. This relation depends on the percentage of other elements, more particularly silicon and the rate of cooling from the molten state, a function of the thickness of section and the temperature of the metal at the time it is poured to a somewhat less degree.

In the absence of silicon, the carbon present remains in the combined form. Hence, commencing with no silicon with dead white and hard iron, there is a regular progression of fractures growing grayer and grayer until at three percent silicon the iron is dead gray and soft. Sudden cooling in a measure prevents this action of silicon, a relation taken advantage of when making chilled rolls, car wheels, crusher jaws, etc.

By securing the proper silicon percentage and the application of iron face moulds, it is possible to get a fairly exact thickness of hard white metal with a soft machinable background.

Manganese is another element always found in cast iron. Below one percent it has but little effect on the character of the casting. Above this it commences to hold the carbon in the combined form. Below one half percent manganese oxidation troubles are liable to be encountered with consequent damage to the casting.

A phosphorus content around one percent secures the fluidity necessary for stove plate and ornamental castings. For the regular run of work, this element should not exceed three quarters percent and for special work it may have to be quite low as in gear castings and the like.

Sulphur is an objectionable and undesirable element in every way for it hardens the casting, making tooling difficult and at the same time weakens it. It is unfortunately always added to a greater or less proportion in cupola melting owing to the proportion of scrap necessarily carried in the daily heats.

From chemical consideration cast iron might be rated as a steel corresponding to the combined carbon present with its graphite content in mechanical admixture.

A cast iron with .25 percent combined carbon, 3½ percent graphite might be looked upon as a .25 percent carbon steel with so much finely divided lubricating graphite within the structure that machining becomes easy and no oil will be required. With .80 percent combined carbon the material corresponds approximately to tool steel in hardness and difficulty of machining.

Dr. Moldenky draws the interesting deduction from this relation that steel additions now so generally used in admixture with the foundary pig represent a reduction in the total carbon with the proper silicon content adjusted so that the combined carbon remains unchanged with much less graphite within the structure. The resulting metal remains machinable to almost the same extent as ordinary cast iron but is much stronger as there is less graphite interspereed in its structure reducing its effective cross section and strength. The product with such admixture of structural steel scrap is called SEMI-STEEL.

A very soft, yet serviceable cast iron may run as low in tensile strength as 12,000 pounds per square inch. Piston rings for automobiles are made so soft that all the wear is taken over by them while again chilled cast grinding mullers high in total carbon are excelled only by manganese steel castings in point of durability.

The lack of homogeneity of cast iron makes tensile tests somewhat uncertain unless considerable care is given the alignment of the specimen in the testing machine. The transverse test is almost universally used, the standard test bar being 1¼″ round cast upright in a dry sand mould. This bar resting on supports 12 inches apart should carry 2,200 pounds central load for machine castings and the like. If some steel has been added, as high carrying capacity as 3,500 pounds centrally applied load will be exhibited.

Compared with steel, gray cast iron is stronger under compression than mild steel but its tensile resistance is only half to one third as great. Its fracture is without material elongation and may be regarded as brittle and unsuited to withstand shock. Hence, a large margin of safety must be allowed where moving parts of machines are made of this material. Its lower cost when compared with steel permits its use in generous proportions.

The phenomena of shrinkage of cast iron must be given attention both in pattern making and design. It causes difficulty in securing sound castings where heavy and light sections join. Thus, the brackets of cast iron columns should not be made heavier than the shell of the column. The shrinkage of white iron castings is about double that for gray iron, the latter being ⅛″ to the foot while white iron contracts about ¼ inch to the foot in cooling.

Combined carbon at high temperature causes an average increase of 100 percent in the shrinkage or contraction of the metal in cooling and as this represents the average or total shrinkage at all temperatures we may conclude from the small difference in coefficient of expansion at ordinary temperature that at higher temperatures this shrinkage may be three to four fold as great with the combined carbon as it is with the carbon in a graphite state or entirely eliminated.

The interesting phenomena observed in the tumbling barrel throws a strong light on the relief of shrinkage strain by shock or range of stress from tension to compression for standard test bars after being rumbled in the tumbling barrel are found to be greatly strengthened by the process—so much so that they are not comparable with the cast metal in the work they are intended to represent.

15. Temper.—Temper is the term employed by the steel manufacturers to indicate the proportion of carbon contained in the steel, (usually a Crucible product.)

Razor Temper.—1½ percent carbon. When properly treated it will do twice the work of ordinary tool steel in dressing chilled rolls.

Saw File Temper.—1⅜ percent carbon. This should not be heated above a cherry red.

Tool and Drill Temper.—1¼ percent carbon.

Spindle Temper.—1⅛ percent carbon.

Chisel Temper.—1 percent carbon. This is well adapted for tools when the unhardened part must bear the blow of the hammer and where a hard cutting edge is required as for cold chisel.

Set Temper.—⅞ percent carbon.

Die Temper.—¾ percent carbon.

The two last may be welded by the mechanic accustomed to weld cast steel. The carbon content may be roughly indicated by the terms low temper, medium temper and high temper steel.

Many manufacturers of tool and high speed steel, differentiate the temper by letters or numerals in their catalogs showing not only the classification but the purpose for which the steel is best fitted.

The term temper indicating the amount of carbon is to be differentiated sharply from tempering or heat treatment of the tool dressor and manufacture.

16. Heat Treatment.—The effects of thermal change upon steel varies greatly with its carbon content and the precise nature of the treatment. Any steel except alloys of silicon, tungsten, etc., may be SOFTENED OR ANNEALED by raising to a read heat and allowing it to be cooled very slowly. When thus annealed, though the carbon content is high, it is relatively soft and may be cut or filed and turned in a lathe.

HARDENING OR TEMPERING tool steel consists of two steps: The finished tool is heated to a bright red, care being taken to have the heat extend back some distance from the edge. The cutting edge is then immersed in water to a slight depth and kept there until it has cooled sufficiently to remain wet when withdrawn. By this treatment the material in the vicinity of the cutting edge has been rendered exceedingly hard but too brittle for service.

The excessive hardness is reduced, and the necessary toughness secured by "tempering" which consists of brightening the metal near the cutting edge by rubbing on emery or carborundum and watching the cleaned area while the heat from the unquenched part extends toward the cutting edge. The oxidation of this brightened surface as the edge becomes hotter and hotter from conduction causes a play of color to become visible which serves as an index of the temperature. The colors run from the hot portion of the tool toward the edge in the following order: Pale yellow, straw yellow, brownish yellow, light purple, dark purple, pale blue and dark blue. When the proper color reaches the cutting edge the piece is again quenched and the tempering is complete.

Turning tools, planer tools, and hammer faces are tempered at pale yellow (430°F.).

Penknives, razors and wood tools, dies, punches, drills, etc., staw yellow (450°F.).

Chisels, wood working tools, plane irons, brown yellow (490°F.)

Light purple, augurs, surgical instruments, cold chisels, etc. (520°F.).

Dark purple, (550°F.) axes, hack saws, springs and wood saws.

17. The Effect of Quenching Soft Steel is illustrated by an experiment by Campbell, given in the following table:

Heat Treatment	Ultimate strength lbs. per sq. in.	Elastic limit lbs. per	Elongation in 8 ins. percent	Reduct ion of area percent	Elastic ratio percent
Natural state; average of 6 bars	46,098	38,825	35.37	70.00	73.37
Chilled at a dull red heat	49,740	33,800	....	70.00	67.95
Chilled at a cherry red	57,240	39,060	....	66.10	68.24
Chilled at a low bright red	58,200	39,930	....	64.80	68.61
Chilled at a bright red	62,640	38,860	....	63.10	62.04

The untreated bar showed the metal as of extreme softness while the chilled specimens proved that each change in the quenching temperature is reflected in the physical condition of the bar. Reduction in area and elogation is somewhat lessened by the treatment.

The effect of combined carbon appears to be to increase greatly the co-efficient of contraction at high temperature. Thus Dr. Moldenke gives the shrinkage of grey cast iron (graphitic carbon largely) ⅛ inch to a foot and of white cast iron (combined carbon) of twice that amount. It would appear that the chilling or sudden contraction of the metal in the outer portion brings intense pressure to bear upon the parts removed from the surface, setting up such a severe state of strain that the metal when quenched at high temperature is exceedingly brittle and toughness is secured only by drawing the temper and reducing the excessive hardness and strain to that re-

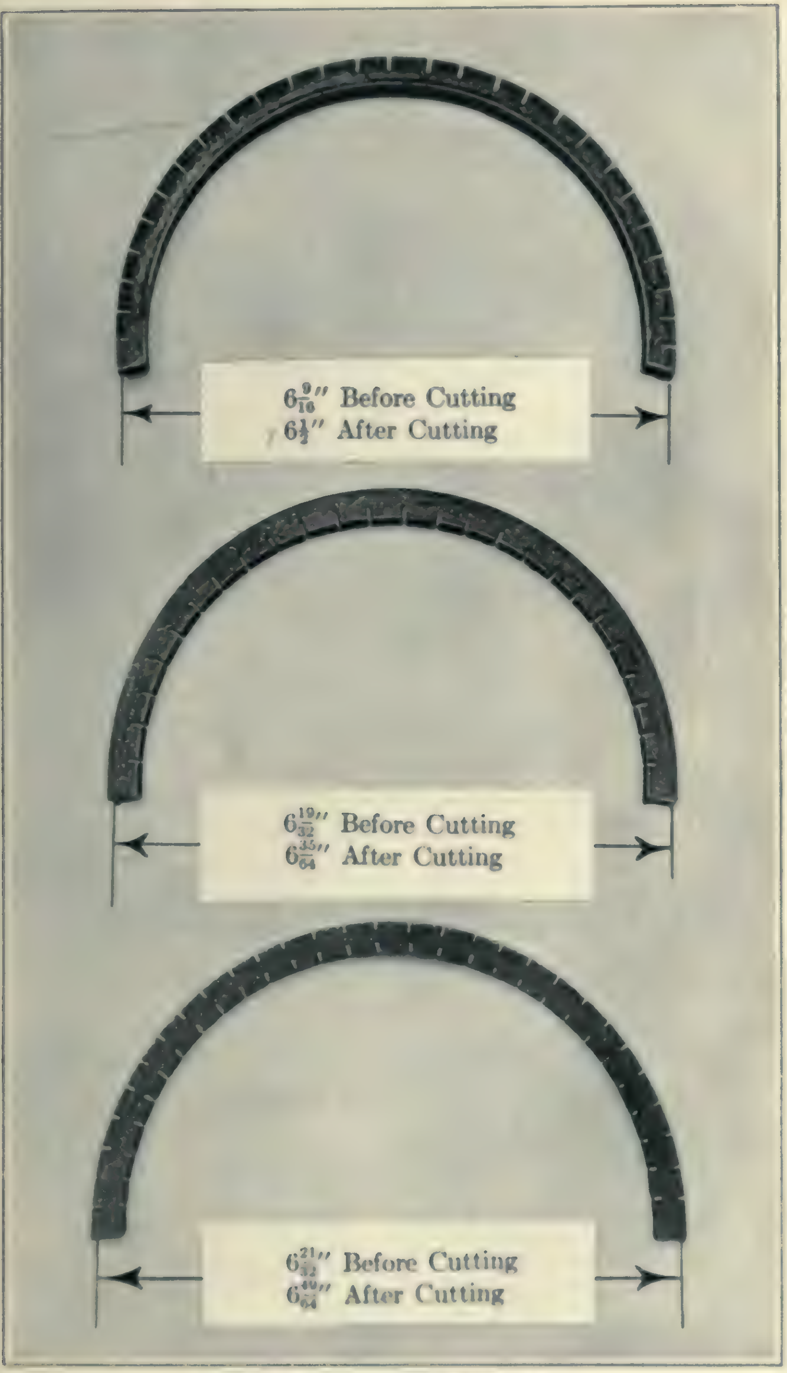

$6\frac{9}{16}''$ Before Cutting
$6\frac{1}{2}''$ After Cutting
$6\frac{19}{32}''$ Before Cutting
$6\frac{35}{64}''$ After Cutting
$6\frac{21}{32}''$ Before Cutting
$6\frac{40}{64}''$ After Cutting

Fig. 33. Experiments With Cold Bent Bars

quired for the purpose for which the tool is to be used. The increased tensile resistance is attained by the reduction of ductility. For example, the very soft specimen, if its ultimate tensile resistance be figured on the greatly reduced area of fracture does not differ greatly from that of the hard steel specimens.

Thus with the reduction in area in the above table of a soft steel specimen of 70 percent, the unit breaking strength per square inch on the reduced area would be between 150,000, and 160,000 pounds per square inch, corresponding to the breaking strength of a steel of high carbon content under Campbell's rule that does not reduce materially in area at fracture.

18. Over Strain and Cold Working.—The softer grades of steel are punched, sheared and bent cold, and the general effect of such cold working upon the material is here to be considered.

Because the softer grades of steel are ductile, or plastic under over stress, they may take a set in one direction or in one portion unaccompanied by corresponding set in another portion. Equilibrium results from a more or less complex state of internal strain, and the elastic conduct of the material thereafter is represented by this state of strain superposed upon the elastic state resulting from the load applied. Its deportment is then said to be aeloltropic or different in different directions.

This characteristic of soft steel is encountered when a key way is cut in a piece of cold rolled shafting. The surface has been symmetrically strained by the work expended upon it by cold rolling. This internal state of stress is thrown out of equilibrium upon removal of the metal in the key way and the shaft then undergoes a slight axial curvature. The following experiment discloses a nature of these reactions.

Cut several pieces from the same $3/8$ inch square mild steel bar, and bend cold to 6-inch diameter as shown in the accompanying figure, so that the spread of the ends is about six inches. Measure out to out spread carefully and record the same. Then score with a saw kerfs about $3/8$ inch spacing on the outside and to the depth of $\frac{3}{16}$ inch. The distance between the ends of the curved bar will elastically decrease $\frac{1}{16}$th of an inch by this process. A similar specimen bent in the same way, sawing kerfs on the inside of the bend will indicate a change of curvature $\frac{3}{64}$th of an inch in the same direction. Another similar specimen treated after bending by sawing kerfs on the outside and inside $1/8$ inch deep only and similar spacing produces a change in curvature in the opposite direction from that of the preceding experiments amounting to $\frac{7}{16}$ths inch spread; a decrease in curvature instead of an increase showing a reaction of elastic strain antipodal to that of the residual strain. The elastic strain being reverse in kind and character to that of the residual strain its effect is to straighten the bar out and reduce its curvature, whereas the residual strain tends to increase its curvature when thrown out of equilibrium. Permanent strain, therefore, is a state of internal stress in equilibrium from which no change in shape results unless this state of equilibrium is disturbed by the action of forces which supply energy from without, or change the area of the resisting section.

Energy changing the internal state of equilibrium may be introduced as follows:

1. By change of temperature:
2. By applied forces:
3. By cutting away a portion of the symetrically strained body.

Such a removal or cutting away of symmetrical portions of an over-strained elastical material permits the permanent strains thrown out of the equilibrium to reassert themselves and change the form of the body in a manner comparable to applied external forces.

19. Aeolotropic Properties of Permanent Strain.—Steel, copper, aluminum, etc., as received from the rolling mills and castings as they come from the foundary are inherently aeolotropic because of the squeeze of the rolls in producing the former, and the shrinkage or cooling stresses in the latter.

The theory of stress intensity and elasticity forming the basis of design assumes their condition as isotropic as an approximate basis of analysis. The investigation of the magnitude of the error of such assumption is now in order.

In the experiment with the bars bent in horse shoe form the spring upon cutting one outer layer was in the reverse direction of the elastic reaction to the forces which caused the permanent bend. This observation is in keeping with that of Joule, that the residual effect of a permanent strain is a change in the position of the metal in the Antimony Bismuth scale of thermo electric potential, in the reverse direction to that caused by the elastic strain which subsists as long as the constraining force is kept applied.

This result may be regarded as a direct consequence of Newton's third law that action and reaction are equal and of opposite sign.

Applied to the case in hand, when applied force is removed and the elastic reaction to the original configuration is prevented by set or plastic deformation along the lines of resultant displacement, the internal stress changes sign.

No over strain has been experienced along the lines of equi-potential and the stress there thrown out of balance is momentarily undiminished and must be brought into equilibrium by residual strain along the lines of displacement following the mechanical law of equality of major and minor stress volumes upon their respective planes of action which is a consequence of the equality of maximum and minimum fiber stress areas treated in Art. 20, Chapter I.

Consider the case of axial tension Fig. 13, Chapter II as the material along *ec*, *ec′* takes a plastic set which opposes elastic recovery along *eo*, *e′o*, the latter elastic stress induces residual counter balancing stress along *ec*, *ec′*. But, since these residual stresses of tension and compression are in equilibrium the axes of the principal stresses of the residual state of strain will shift to a position at 45 degrees to the axis of the bar.

The effect of superposition of such a state of residual strain upon an isotropic material would be to increase the Y (yield point) limit of plastic set, reduce somewhat the value of the modulus E and the coefficient K.

The limit of elastic stress along the equi-potential curves within the elastic range of stress along the curves of displacement would limit the reduction in the value of K from an isotropic state (.4142) to .336. This is above the value of .3 of the ordinary test as it should be when the restraint of the grips is considered, therefore we may rationally consider the state of residual strain from rolling the limit of plastic deformation.

The elastic unit stress along the curve of equi-potential equals half that along the curve of displacement, hence its momentary value at the instant of plastic yielding on a 45 degree line equals $P\ cos. \times sin\ 22°30'$ which is out of balance. Equilibrium will accordingly require a static balance of half the stress intensity by an equal and opposite stress at right angles, in the resultant state of residual strain. This, with respect to elastic elongation under a further load is equivalent to an applied force of the opposite kind. On this basis K equals .336, F equals $.37E$, the value of E is approximately $29 \times (10)^6$ for soft steel and $3 \times (10)^7$ for hard steel.

SUMMARY: The foregoing discussion of the properties of materials is of necessity meager, and logically limited to those phases closely related to their elasticity and resistance. An attempt has been made to present the outstanding facts regarding conditions, leading to toughness and dependability of timber in respect to age and condition of cutting and to explain the reason for and necessity of limitation of impurities in steel and the outstanding phases of its manufacture, heat treatment and condition after over strain. These matters are commonly not understood at all by the graduate of our civil engineering schools.

The short discussion of steel here presented may best be supplemented by referring the student to The A. B. C. of Iron and Steel, Penton Publishing Company, Cleveland. Howe's Metallurgy of Steel and Manufacture and Properties of Structural Steel by H. H. Campbell.

A fairly complete, treatment of concrete, discussion of aggregate, mixture, handling at different temperatures, shrinkage, strains, etc., will be found in Concrete Steel Construction, by Eddy and Turner.

CHAPTER V.

INVESTIGATION OF THE RANGE OF IMPERFECT ELASTICITY WITHIN THE YIELD POINT VALUE OF STEEL, ENDURANCE OR FATIGUE TESTS, AND THE WORK OF RUPTURE.

1. Where an elastic body is subjected to stress, and the strains it undergoes do not pass beyond the limits of the true elasticity of the materials, the work expended in straining the body is restored by its reaction as the stress is removed. But when the strains exceed those limits, work is expended upon the material that is not so restored. The energy so expended beyond the elastic resilience of the body is dissipated through the evolution of heat, and the body is not able to restore fully the energy imparted to it by work performed beyond the true elastic limits.

This work done upon the material tends to change its properties. When a piece of mild steel is gradually loaded in simple tension to between five eights and two thirds of its ultimate strength, a point is reached where a large plastic deformation occurs with very little increase in the applied load. This is called the yield point, or commercial elastic limit, and is determined by the drop of the beam in testing. When the load is carried to a point somewhat beyond this primitive yield point and the load is then removed, on subsequent testing of the same piece it will be found to have developed a new yield point at a somewhat higher load than that previously found, while the limit of proportionality between stress and strain has at the same time been reduced below the value previously found. If, after being considerably over strained the specimen be allowed to rest for a certain interval of time before being reloaded, it then exhibits greater hardness than does a similar specimen similarly treated and reloaded immediately. Its yield point is further raised by such a period of rest and its elongation at fracture is reduced.

The apparent limit of proportionality rises during this period of rest from that position to which it has been lowered by the over strain. When a piece of steel is strained beyond the limit of true proportionality and within the yield point value, a temporary strain is observed after the removal of the load. The disappearance of the temporary strain which occurs by a slow creeping back of the material is called elastic after working and the lag or slow increase of deformation under stress is called hysteresis.

2. Elastic After Working.—The German writers refer to this phenomena as "elastische Nachwirkung," which might be translated "elastic reaction" if the word reaction were not already used in a different sense. The phenomena is more readily observed by twisting a fine wire suspended from a fixed support, having a small mirror attached to the lower end, the position of which can be observed in the usual way by means of a telescope and scale. If the lower end of the wire is twisted through an angle not too great and then left to itself, the mirror makes oscillations, the extent of which may be read off on the scale. These oscillations decay more rapidly than as if the only retarding force were the resistance of the air, showing that the force of torsion in the wire must be greater when the twist is increased than when it is diminishing.

"If we begin by keeping the wire twisted (at not too great or too small an angle) say for a minute or an hour, and then leave it to itself, we find that the point of temporary equilibrium is displaced in the direction of twisting, and that this displacement is greater the longer the wire has been kept twisted. But this displacement of the point of equilibrium is not of the nature of a permanent set, for the wire, if left to itself, creeps back towards its original position, but always slower and slower. This slow motion has been observed by the writer going on for more than a week, and he also found that if the wire was set in vibration the motion of the point of equilibrium was more rapid than when the wire was not in vibration.

We may produce a very complicated series of motions of the lower end of the wire by previously subjecting the wire to a series of twists. For instance, we may first twist it in the positive direction, and keep it twisted for a day, then in the negative direction for an hour, and then in the positive direction for a minute. When the wire is left to itself, the displacement, at first positive, becomes negative in a few seconds, and this negative displacement increases for some time. It then diminishes, and the displacement becomes positive and lasts a longer time till it too finally dies away." (Maxwell, Enc. Brit. Constitution of Bodies.)

The phenomena above described represents an angle of distortion greater than the true elastic limit but less than the yield point of the metal.

Kelvin has treated this phenomena as the "viscosity" of elastic solids.

In the preceding discussion this so called viscosity has been explained by the fact that there must be transferred from the internal heat energy of the material to potential energy of strain double the mechanical work in equivalent heat units to that expended in producing the strain. This should be obvious since the effect upon the heat vibration of the particles composing a body is a function of the final state of deformation and independent of intermediate states, whereas the static external work performed is per contra a function of the intermediate states rather than the final state which is opposed and held in equilibrium by the internal strain. In other words, the external mechanical work is half the product of the final force multiplied by the strain or displacement. The thermo dynamic effect is a function solely of the total displacement, hence twice the mechanical work causing the given displacement.

2. Apparent Elastic Fatigue. Experiments on the torsional vibration of wires detailed by Kelvin in his article on Elasticity in the Encyclopedia Britannica, indicate elastic fatigue for a considerable range of stresses through which the experiments were conducted, this range being within the limit of elasticity of shape. The experiments were as follows:

About April 26th, two equal and similar lead weights or vibrators were hung up by equal and similar copper wires, the upper and lower ends of the two wires being similarly fixed by soldering. Wire No. 2 was kept vibrating as continuously as possible from day to day, and the other kept at rest except when vibrated once a day in an experiment to test the comparative vicosity of the wires, but no exact comparison was made until May 15. Then No. 1 in 97 vibrations subsided from an initial range of 20 to a range of 10, while No. 2 gave the same subsidence in 98 vibrations. During the greater part of May 16 and 17, No. 2, was kept vibrating and No. 1 quiet and experiments made, with the result that No. 1 subsided from an initial range of 20 to 10 in 98 to 99 vibrations of 2.4 seconds each, while No. 2 gave the same subsidence in from 57 to 60 vibrations of 2.4 seconds each.

It was supposed from these experiments that true elastic fatigue occurred for all stresses within the limit of elasticity of shape until a similar and more extensive series of experiments conducted by H.

Dissipation of energy by heat in the non-reversible cycle of imperfectly elastic strain beyond the true limits of elasticy is the explanatiion of the apparent fatique above noted.

Thomlinson (Phil. Trans. Royal Soc. 1886) showed that no such fatigue of elasticity is felt in steel, iron or copper provided the stress does not exceed a certain well defined limit.

4. Variation in Yield Point Values of Mild Steel.—In Art. 1, it is noted that by straining the material above its primitive yield point, releasing the load and reapplying it a yield point or drop of the beam will be observed at a somewhat higher value. Again, when a bar of steel is loaded a number of times alternately in tension and compression to a point a little beyond the primitive yield point values in tension and compression, these values are thereby both reduced to about .6 their original values called by Bauschinger the NATURAL ELASTIC LIMITS of the material.

When the material is repeatedly strained in tension from zero to a value above the natural elastic limit in tension but within the yield point, the yield point value in tensoin is raised by such treatment and in like manner when similar stresses above these natural elastic limits but within the yield point values, are applied alternately from tension to compression, the primitive yield point value in tension is lowered. This phenomena raises the question whether the temporary strain which accounts for the elastic after working is of the same nature as that produced by a permanent set and what the limits of its accumulation may be under repeated stress in working of the material without giving it an opportunity to recover by elastic after working between consecutive loadings.

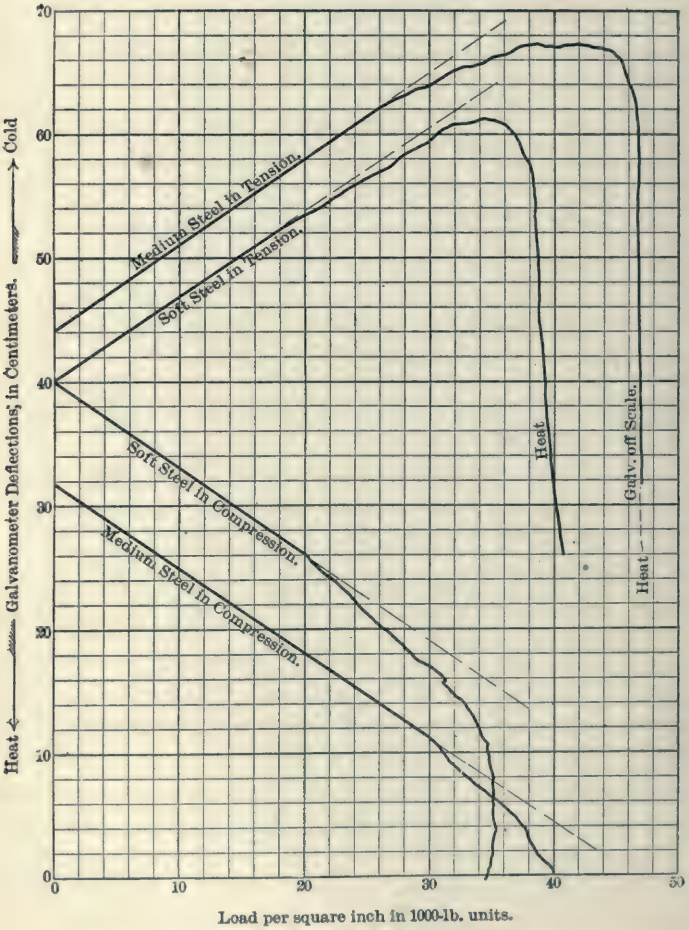
These questions would seemingly be more readily investigated from consideration of the thermal stress curves presented by the material under load.

Fig *A*, shows typical thermal stress curves for soft and medium steel in tension and compression, given by the author in Trans. Am. Soc. C. E. 1902.

It will be noticed that in tension that the soft steel grows regularly cooler until about five eights the yield point value is reached when the heat of imperfectly elastic deformation causes the curve to depart from the straight line relation until the yield point is reached when there is a sudden evolution of heat. In like manner, under compression the straight line relation is maintained until baout five eighths the yield point value is reached, then a similar phenomena occurs.

The identity of cause, i.e., thermal effect would indicate identity of result. In other words, within the range of imperfect elasticity within the yield point under repeated working of the material beyond its natural elastic limits, an accumulation of residual strain occurs with each repetition of the applied load even though no apparent set or plastic deformation is noticeable. As the apparent yield point is gradually raised by such treatment, the range of stress may be gradually increased to a higher and higher amount until the ultimate resistance of the material is reached without noticeable plastic deformation of the same piece of steel which under a single application of the load would stretch out twenty percent of its length before necking down and final fracture.

TYPICAL THERMAL STRESS CURVES.



The fact that a residual strain is of the opposite sign to the internal stress opposing the force which produced it leads at once to the question whether change in the yield point value is a phenomena of residual strain or change in structure. It has been found impossible to discover microscopically a change in the nature of the material which has undergone a sufficient number of repetitions above the natural elastic limits of the material to nearly wear it out or cause it to approach the point of rupture.

5. **Woehler Experiments.**—The behavior of iron and steel under flucutating or repeated stresses was the subject of long research by Woehler and later by Bauschinger. The result of these experiments are claimed to show:

1. That resistance to fracture under stresses fluctuating within certain limits depend upon the difference between the maximum and minimum applied stresses rather than upon the maximum stress alone.

2. And that reversed stresses, tensile and compressive, much below the static breaking stress, are sufficient to cause fracture if repeated a great number of times.

Experiments upon axle iron made by the Phoenix Company, having an ultimate strength in tension of 23 tons with an elongation of about 20 percent, gave limiting maximum stresses under limited ranges as follows:

Kind of Repeated Load	Stress in tons per square inch	
	Limiting Maximum	Limiting Range
Completely reversed	7.6	15.2
From Maximum to zero	15.28	15.28
From Maximum to half load	21.01	about 10

The number of repetitions of stress upon the specimen, still unbroken, from which these limiting ranges were determined, ran into the millions. The specimens unbroken at an alternating stress of 7.6 tons were subjected to 132,000,000 repetitions without breaking. This limiting range which allows nothing for dynamic effect is nearly .6 of the primitive yield point value of the material, and if the dynamic effect of the rapidity of repetition of stress amounts to 10% the limiting range would be in the neighborhood of ⅔ of the primitive yield point.

Now the ultimate strength of the material was 23 tons and its primitive yield point was in the neighborhood of 12 tons.

The limiting maximum stress of 21 tons for repetitions of half load in the above Table is therefore at least 9 tons above the primitive yield point, and since whenever the primitive yield point is exceeded a large deformation results, the manner in which these repeated stresses were applied is of interest.

To avoid the large deformation which would have been produced when the primitive yield point is exceeded, the stresses first applied in these experiments, were less than the primitive yield point and this point was thus gradually raised until its ultimate position was above the maximum stress considered in the range noted as the limiting range.

In a practical structure the conditions of loading are not such as to raise the yield point value of the material and so increase its resistance, as was done in these experiments.

Consequently, the limiting ranges of intensity of stress which the practical structure can withstand cannot be deduced from experimental results where such artificial assistance is extended to the resistance of the material.

A dynamic theory of fatigue has been propounded by T. C. Fidler, in which the breaking load "a" is derived from the applied stress "t" as follows:

1. Steady load no variation........................$a = t$.
2. Load varying from O to t ∓ ∓......................$a = u = t/2$.
3. Load varying from +T to —T......................$a = u = t/3$.

As further experiments have shown that the number of repetitions of stress beyond Wohler's limiting ranges required to produce rupture is substantially constant whether the speed of application of the fluctuating stresses is decreased to half that in Wohler's work or doubled. Moreover, while the effect of a load imposed instantaneously and without shock, causes momentarily twice the effect of an equal gradually applied load in producing deflection and strain, it is illogical to suppose such a relation could possibly exist with the time of application extended thru the interval of fifteen seconds as was the rate of application of repeated loads in Wohler's experiments. Moreover, if the method of computation be applied to the loading within the limiting ranges of tension to compression it at once appears that the computed stresses have exceeded the yield point by 50 to 60 percent without raising it at all. These two difficulties render the theory wholly untenable.

The micro flaw theory of fatigue advanced in Johnson's "Materials of Construction," is equally untenable in its failure to account for observed phenomena. Thus, the elastic fatigue in Thompson's experiments depending upon the immediately previous molecular condition whether of quiescence or recurring changes of shape cannot be accounted for by micro flaws which are necessarily permanent and not temporary.

As the originators and advocates of the fatigue formulae of proportioning agree that if it be established that Wohler's results are accounted for by the effects of stress beyond the true limits of elasticity they have no relation to the logical determination of working stresses, this matter will now be considered from the standpoint of the three determinants of ideal elasticity, to-wit:

1. Proportionality of temperature change to the load applied:
2. Proportionality of elastic elongation to the applied stress or load:
3. The reversible cycle of mechanical work during loading and unloading.

No bar of sound metal has ever been broken by Wohler's treatment under ranges of stress of one kind or of both kinds, i.e., from tension to compression within the thermal limits of proportionality.

Per contra range of stress from tension to compression within the natural elastic limits was found by Bauschinger to improve the metal in elasticity, tenacity, and ductility instead of injuring it.

The radical improvement in the strength of cast test bars and the suggestion that they shall not be rumbled if their modulus of rupture be taken to represent the average condition of the casting, is a striking confirmation of the beneficial effect of stress ranging from tension to compression in reducing and eliminating residual states of stress arising from rolling or shrinkage in cooling.

The theory of residual strain shows that the range of stress of one kind is not as efficient in eliminating residual strain as range of stress from tension to compression. Where the range is from tension to compression extending equally above the natural elastic limits for tension and compression the reaction of the accumulation of residual strain is neutralized because of the opposite signs of its constitutents accompanied by increased internal disturbance and more rapid approach to rupture under repeated stress. Where the limiting range was tension from—10 to —21 tons the systematic work upon the specimen has effected an accumulation of approximately +9.5 tons residual strain which added to the primitive yield point value of 12 tons would give a maximum resistance of 21.5 tons without plastic deformation. In other words, the effect of a tensile stress of —21 tons on a bar under a state of residual tension of +9.5 tons or more could not cause plastic deformation in a specimen whose yield point is 12 tons; since the algebraic sum of —21 +9.5 is but —11.5 therefore less than the yield point of the bar.

The fact that rupture of ductile metal under Wohler's treatment takes place without plastic elongation has been in the past erroneously regarded as in some way indicating or establishing the proposition that all ranges of stress without shock tend to deteriorate or injure sound metal.

6. Work of Rupture.—The law of conservation of energy indicates that the same amount of energy must be expended to produce rupture of the same cross section of a given kind of material whether it is expanded in small quantities at different times or at one time in a single application.

Since no portion of the work of rupture can be stored within the metal, it must be transformed into heat readily measured by thermo electric means. Such methods of investigation treated by the writer in Trans. Am. Soc. C. E., August 1902, furnish a very precise and sensitive method of determining directly the true elastic limit of the metal. The deviation of the thermal stress curve from linear proportionality before the yield point is reached, represents the mechanical work expended in permanently changing the state of strain in the structure of the metal.

Where a bar of steel is ruptured by a single application of load gradually increasing from zero to its ultimate limit resistance, but a small portion of the deformation is elastic, viz., that part which occurs before the yield point is exceeded. Permanent stretching then commences and continues until the bar if mild steel may have elongated twenty percent of its original length. The mean stress causing plastic deformation would be a mean between the yield point and its ultimate resistance or about 50,000 pounds per square inch, consequently the work absorbed per cubic unit of metal before reaching the point of incipient rupture would be 50,000 × .2 or 10,000 inch pounds of work disregarding the elastic work which is relatively negligible. Now 10,000 inch pounds of work by Joule's equivalent equals 1.07 times the heat required to raise a pound of water one degree F. But this amount of heat is sufficient to raise the temperature of the bar 33 degrees F an amount readily noticed by the touch or feeling of the piece.

When rupture occurs accompanied by nearly negligible ultimate deformation as in endurance tests under repeated stresses of intensity lower than the ultimate carrying strength of the material there appears no good reason to suppose that the total energy transformed into heat in small amounts under successive repetitions of the load in endurance tests, is any less than where this deformation is as large as it is in the case of a single load which develops the maximum carrying strength of the specimen.

In case of hard steel, since the natural elastic limit is higher the limiting range of fluctuation stress from tension to compression is higher than for soft steel.

But the endurance of the harder steel beyond these limits is much less than that exhibited by softer metal for two reasons:

1. More heat is generated in hard steel when the natural elastic limits are exceeded than is the case in the soft steel specimen; hence more work on the structure with each repetition.

2. The total work of rupture is less because the ducility and hence elongation when tested in the usual way is less than for softer steel.

These deductions from the work of rupture and thermal curves are fully confirmed by the experiments of Wohler and Bauschinger.

7. Shear Lines.—If the side of a flat bar of soft steel be polished and then strained up to the yield point diagonal markings appear on the polished surface, called shear lines, as shown in Fig. 16.

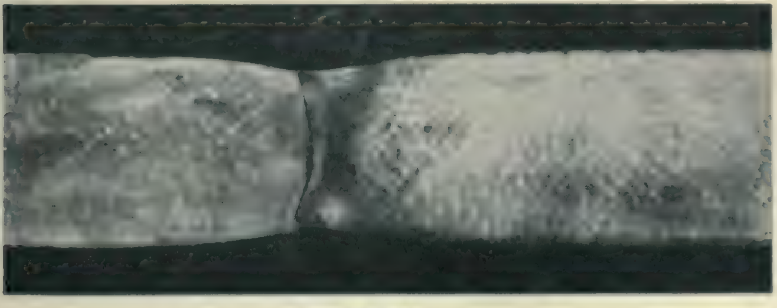


Fig. 16
Showing Shear Lines Developed in Tensile Test of Steel Bar Beyond Yield Point.

Their direction differs somewhat from 45° determined as follows from the principles established in Chapter I. The plane of incipient failure from direct stress would tend to bisect the angle between planes normal to the curve of displacement and the line of greatest shear. Hence were the steel free from residual strain from rolling these lines would be at 56°15′ to the axis of the bar and the plane of slip normal thereto at an angle of 33°43′. For K = .336 as the condition of the material as received from the mill theses angles would shift 2°45′ nearer to symmetry with the axis or the marking would assume a position of 53°30′ and 36°30′ to the axis respectively, as measured on the specimen of Fig. B. As K has a different value for hard steel the slip lines will alter their angle accordingly. Such markings have been miscalled shear lines or stress lines (Martens) notwithstanding their direction is a derivative only of the lines of displacement and shear. Actual or final fracture usually takes place normal to the principal tensile stress.

8. Practical Aspect of the So-Called Factor of Safety and Maxima and Minima Formula of Proportioning.—Weyrauch and Launhardt formulated an algebraic method to allow for the supposed injurious effect of range of stress without shock. The derivation of these formulas in principle is analogous to the old idea of a factor of safety based upon the ultimate carrying strength of the metal in a testing machine rather than what it can carry on the job.

To illustrate the point, a piece of mild steel has a tensile breaking strength in a machine (which does not follow up plastic deformation as a load acted upon by gravity would do) of say 64,000 lbs. per square inch. If the so called factor of safety is four 64,000 lbs. divided by four equals the working stress equals 16,000 lbs. per square inch.

Suppose the load to be suspended by a short hanger and increased to 40,000 lbs. The yield point has then been exceeded and plastic yielding is followed up by the free downward motion of the load. The dynamic effect would approach that of a suddenly applied load sufficient to rupture the hanger at five eighths of its laboratory carrying strength under test.

The true factor was thus about two and a half instead of four. It may be objected that there are many examples of partial bridge failure where the post buckled out of shape without collapse or cases where a counter after noticeably stretching did not cause failure. The answer lies in the fact that railroad bridge loading is a rapidly moving and fluctuating load, before the member has had time to be deformed materially the stress has greatly decreased—a condition vastly different from the case of static suspended load under discussion. Likewise, in a frame or truss, large plastic deformation of a member might so change the triangulation that here also, the yield point rather than the testing machine ultimate, limits the margin of safety.

The old factor of safety soon came to be classified as a factor of ignorance because unrelated to the true margin of safety as it is understood by the engineer.

The general form of maxim and minima formula is given in the 18th edition of Trautwine—

$$(1)\quad a = u\left(1 + \frac{\text{min. stress}}{\text{max. stress}}\right) \text{ range of one kind}$$

$$(2)\quad a = u\left(1 - \frac{\text{min. stress}}{\text{2 max. stress}}\right) \text{ range of both kinds}$$

u was commonly taken as 6500 for wrought iron bridges.

When there is no variation $a = 2u$.

When $+$ min. stress $= -$ max. stress $a = u/2$.

It thus appears that the variation in the working stress was the liberal amount of three hundred percent of an imaginary effect.

The derivation of these formulas is as unscientific in conception as the factor of safety, unrelated to the true margin of safety on the job but in addition their application to have any bearing whatever upon design assumes conceivable kind of loading impossible in practice, to-wit, systematic and precise working of the metal to acquire that remarkable accumulation of residual strain produced in the laboratory alone and impossible elsewhere. The fact that heat energy of inelastic deformation can be measured on all ranges of stress higher than the natural limits of elasticity bars their consideration from rational practice in design.

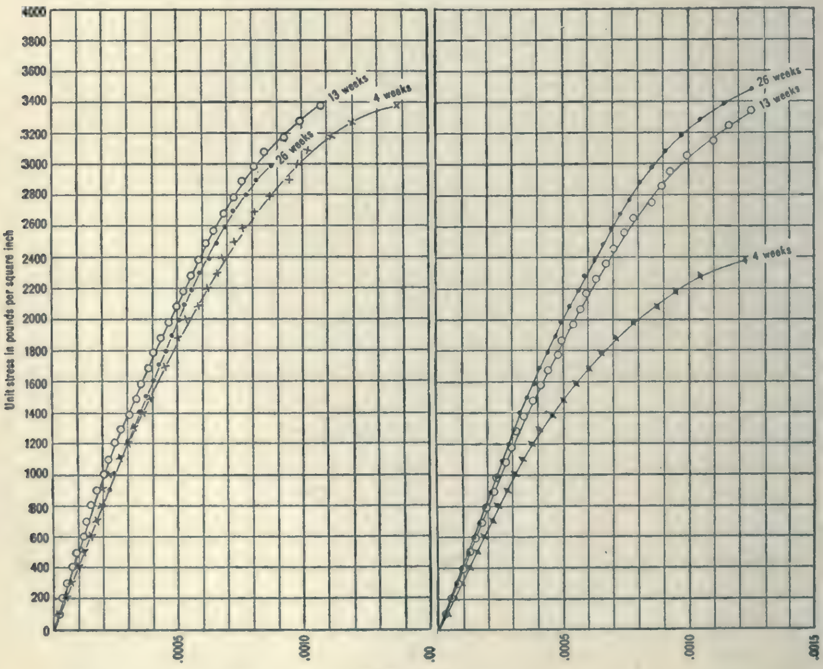
These formulas give rise to unbalanced distribution, to efforts to economize by using dangerously high working stresses where the range is small and to excessive waste when the range of stress from tension to compression would tend to improve rather than injure the material. Such formulas are being gradually relegated to the junk heap by accumulated experience. Nevertheless, they are still taught the student of machine design as though in some manner they might be of assistance in proportioning machine parts subject to hard usage and repeated shock, in place of practical observance and study of results of service of similar parts under similar or analogous conditions and proportioning in the light of known facts rather than by algebraic formulas wholly inapplicable thru lack of analogy to conditions they were intended to represent and the erroneous and unscientific conceptions upon which they are founded. They may therefore be dismissed as useless; productive of confusion and waste rather than rationally balanced design.

CHAPTER VI.

WORKING STRESSES DEDUCED FROM ELASTIC RELATIONS AND DYNAMIC EFFECTS IN LOADING

1. **A Stress Strain Curve** represents the elastic reactions of a specimen of material under axial loading by plotting the unit intensity of the applied loads as ordinates and the deformations as absissas at some convenient scale. Study of the deviation from lineal proportionality of ideal elasticity is facilitated by this method of presenting the results.

Figs. 17 and 18 show typical stress strain curves of concrete. The curve clearly departs from the straight line relation even for loads less than half the ultimate. But if after loading to about half or a third ultimate the load be removed upon repeated reloading the curve approaches more and more nearly the straight line relation. Cast iron presents the same characteritic.



Figs. 17-18.

Deformation Per Unit of Length
Characteristic compression-stress deformation diagrams, granite concrete of medium consistency; ages 4, 13, and 26 weeks.

Deformation Per Unit of Length
Characteristic compression-stress deformation diagrams, limestone concrete of medium consistency; ages 4, 13, and 26 weeks.

When steel is tested in commercial work, the weight is run out on the scale beam as the load is applied so that the beam floats or swings free. The strain increases substantially as the applied load to half the ultimate resistance; then at slightly more rapid rate to about five eights of the ultimate resistance when a sudden plastic yielding occurs and the scale beam drops. This is the primitive yield point of the material, and the commercial elastic limit as it is unscientifically termed.

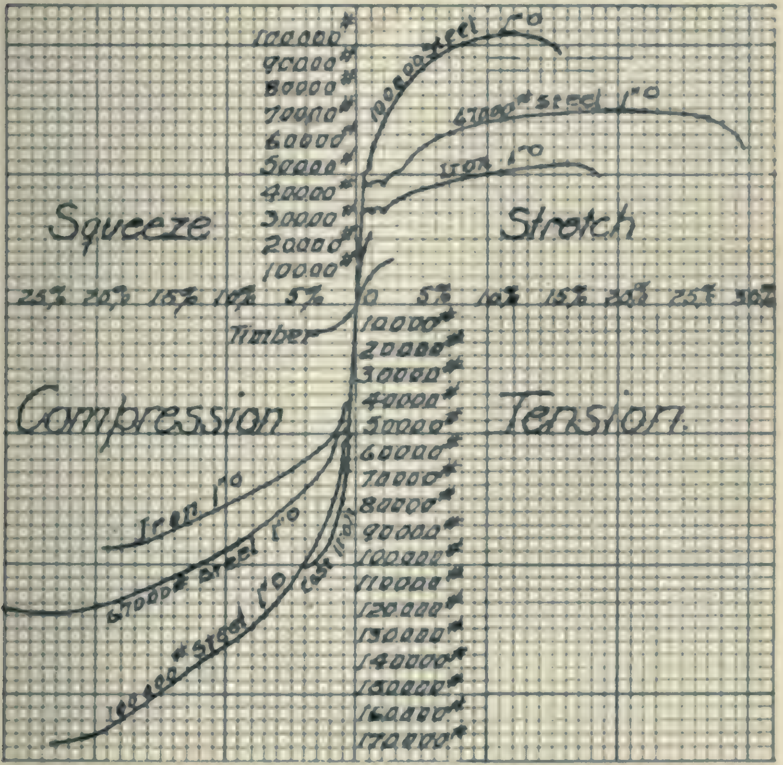


Fig. 19

Mild steels carbon .20 to .40 are tough and ductile, stretching from 28% for .20 C to 18 to 20% for C = .40.

An illustrative diagram exhibiting the typical deportment of hard and soft steel, timber, wrought iron and cast iron under tension and compression and the elongation or shortening before failure is presented in Fig. 19.

Fig. 19 shows the stress strain curve of a cast iron specimen, lacking in the straight line proportionality of strain to the applied load presented by the structural steel specimen because of shrinkage stresses in cooling. When such a specimen is stressed to three eights its ultimate resistance and the load removed and the process repeated a few times the initial curvature tends to disappear and to approach more and more closely to a straight line relation.

Stress strain curves of concrete in compression exhibit like characteristics indicating that under initial loads in general slight set or permanent strain occurs. The material tending to become more and more perfectly elastic as the material is loaded and unloaded a number of times.

This phenomena in cast iron as shown by the increase in the modulus of rupture by rumbling the specimens. The initial application of load upon a specimen of rolled steel or wrought iron also will present noticeable irregularities under precise methods of testing, indicating the presence of permanent strain from the quenching effect upon the hot metal of the water for cooling the rolls.

The residual strain from the stretch and squeeze of the rolls would not be disclosed by saw kerfs as in the case of the bent bar, since such effects are symmetrically and uniformly distributed.

Cutting saw kerfs in a fifteen inch long 5/8 inch square or round bar, half inch spacing, $\frac{3}{16}$ths inch deep on either the top or bottom of the bar as it passed thru the rolls, no noticable change in curvature becomes apparent.

On the other hand, cutting the kerfs on one side of the bar as it passed thru the rolls from a thirty second to a sixteenth inch in curvature from the initial straight line form results, a measure of the permanent strain of the quenching effects of the water cooling the rolls. Fluctuating stresses of moderate intensity, vibration, slight shock and the expansion and contraction of temperature changes all tend in time to reduce or eliminate such states of permanent strain and improve the elasticity of the material.

Cold rolling the mild steel shell plates of a boiler sets up a state of residual strain which is so relieved by temperature changes that no deleterious effect is found in the subsequent deportment of the plate.

2. Aeolotropic Modification of Naturally Isotropic Properties in Metals.—From the discussion in Art. 1, it appears—

(*1*) That the naturally isotropic characteristics of the metals of construction are changed to aeolotropic to some extent by residual strains of cooling and working in course of manufacture:

(*2*) That in general these states of residual strain are of minor rather than major magnitude.

(*3*) That repeated stress or vibration well within the natural limits of elastic deportment tend to eliminate this state of residual strain with resultant improvement of the metal.

What then is this improvement in terms of numerical effect upon the coefficients, E, K, and F? Bauschinger's work demonstrates that E is increased numerically. The principle developed in Chapter III that volume is a kinetic invarient of elastic strain establishes the fact that the coefficient K for mild steel increases from .336 (the usual value of .3 corrected for the restraint of the grip) to .4142 for true isotropy. The relation of the coefficients, $1 + K = \frac{E}{2F}$ shows that F in terms of E decreases as K increases. But since the state of residual strain is a state of stable equilibrium until disturbed by energy from without F representing rotational strain must be unaffected numerically. Therefore when F is calculated from the experimental values of K and E of the material under residual strain, the numerical value of E when the residual strain is dissipated is at once known because F is numerically constant. When the numerical value of E experimentally determined is 29,000,000 and $K = .336$ for a piece of soft steel, $F = .378\ E$. When K approaches its isotropic value of .4142 +, F approaches .3535 E, F being numerically constant it appears that the isotropic value of E is nearly 30,500,000 and that the ultimate improvement in elasticity produced by range of stress within the natural elastic limits is an increase in E from 29,000,000 to an extreme value of 30,500,000. Thus the elongation under a given pull may be reduced one twentieth and the lateral contraction increased a corresponding amount by dissipation of residual strain nduced by the rolling process.

3. Thermo Elastic Relation Between the Sign of the Stress and Temperature Change in the Material a Constant.—Reasoning from cause to effect tension in the material should cause all elastic bodies to grow cooler and compression warmer.* Apparent but not actual exceptions to the logical sequence of cause and effect are presented by India rubber and certain materials of cellular structure. Such material as India rubber is not uniform but composite, e. g., presenting the characteristics of the rubber superposed upon the properties of the gas or air filling the cells of the material.

If a rubber band be stretched the heat generated can readily be noticed by touching the band to the lips. If a weight be suspended by an elastic band and a hot poker be brought near the band it suddenly contracts and raises the weight. Suppose the air cells to be spherical before loading them for a very small strain; they will become ellipsoids of approximately the same volume as the spherical cells and the cooling effect may be measured by thermo electric means.

For a relatively large strain, however, the cells are flattened out and their volume reduced; hence the air or gas in the cell becomes compressed and heated by this compression an amount far greater than the cooling effect of stretching the rubber, completely over shadowing that effect. Likewise, the expansion of the air filled cells produces excessive lateral bulging accounting for raising the weight when the heated poker is brought along side of the rubber. Thus, the validity of deduction from the sequence of cause and effect is substantiated rather than refuted when the diverse elastic reactions of the different parts of the composite material are given proper consideration.

THERMAL STRESS CURVES

BEFORE AND AFTER OVER-STRAIN.

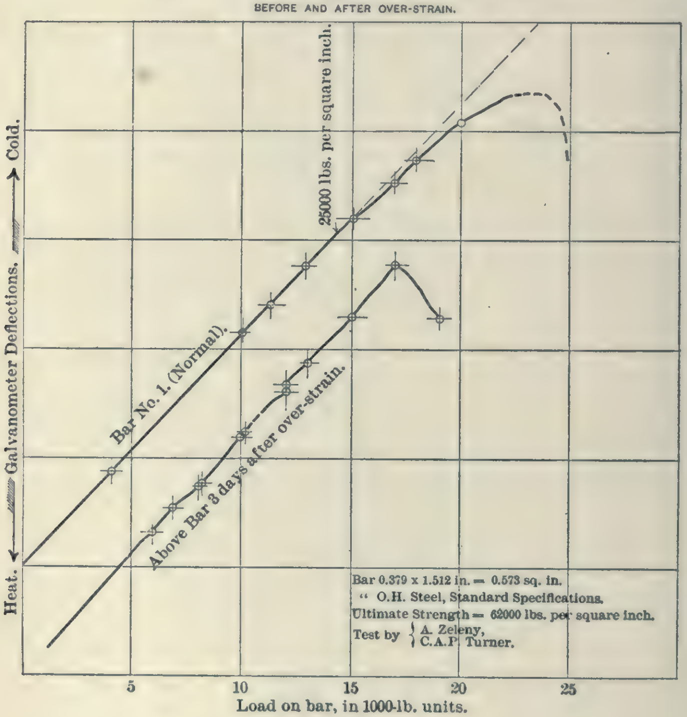


Fig. 2

4. Relief and Accumulation of Residual Strains: Permanent strain is relieved by range of stress within the natural limits of elasticity but accumulates when the range of stress exceeds these limits as shown by the thermal stress curve of a bar three days after over strain as determined by the author is reproduced from Trans. Am. Soc. C. E., August 1902. The irregularity of the curve indicates the dissipation of energy in non-elastic resistance in proportion to the range of stress therein agreeing with Wohler's conclusion regarding the effect of stresses exceeding these limits.

5. Modification of Volume by Residual Strain.—Stokes, in his mathematical and physical papers (1845 Vol. 1, page 75, points out that the modulus F and the bulk modulus B are of basic importance in theoretical work while in engineering work on the other hand E and F are more convenient. We have reduced the kinetic and geometric moduli of the four coefficients of elasticity E, F, K and B regarded heretofore as partly independent, to correlated and dependent values in which B is unity for the kinetic modulus of elastic stress in the uniform isotropic solid. As before noted, B differs from unity for its static value. Its modification in states of residual instead of elastic strain will now be considered.

Cold working, such as stretching the material or compressing it so that it bulges laterally, opens the grain and reduces the density. Annealing on the contrary increases the density, quenching reduces the density which is again increased from the chilled hard state by drawing the temper and again quenching. Coefficients of expansion are modified by these changes.

It appears that the effect of quenching is that of an extreme condition of residual strain in which the enormous pressure of the chilled exterior, while perhaps closing the grain of the interior, stretches and opens the grain of the exterior portions. Annealing, drawing the temper to the desired point, and quenching, by relieving the intense strain of the exterior portion, changes the properties of the metal from brittleness to toughness, reduces the hardness which is even then sufficient for edge tool purposes and renders the tools reliable in that they can perform their work without breaking under shock or jar.

An interesting field of investigation is here open, experimental work along this line having heretofore been somewhat fragmentary.

6. Dynamic Effects.—When a load is applied suddenly without shock the momentary effect of that load is to produce a deflection or strain twice as great as the same load gradually applied. Accordingly, in bridge structures the suddeness of application is provided for under what is customarily termed impact allowance.

The additional deformation of a member under load applied with complete suddeness over and above the static deformation depends upon the magnitude of dead load stress on the member or it is proportional to $\frac{1}{D+L}$ in which L = live load stress, and D = dead load stress.

The proportional relation of the loading to complete suddeness will be an inverse function of the speed of the train relative to the length of loaded track A, which produces the maximum stress (S). Thus, if the maximum speed is 100 feet per second, then for vertical vibration the impact would be approximately $\frac{100}{A+100}$ for complete suddeness, considering the element of time of transmission through the elastic ties or floor metal.

The approximate total vertical impact $I=S\left(\frac{\mathrm{L}}{D+L}\right)$.

The rolling load tends to set up structural vibrations laterally as well as vertically; consequently the total impact is the resultant of the vertical and horizontal effects combined. In turn, the stiffness of the truss articulation enters as a factor so that we may write as a final approximate value

$$I=S\left(\frac{\mathrm{L}}{D+L}\times\frac{140}{A+100}\right) \text{ for pin connected railroad bridges;}$$

$$I'=S\left(\frac{\mathrm{L}}{D+L}\times\frac{130}{A+100}\right) \text{ for riveted railroad bridges:}$$

Where the range of stress is from compression to tension or vice versa, L must overcome D of the opposite sign in reaching its value; hence the change in stress is $L+D$ for the numerator and the first factor becomes unity, and I in per cent of S becomes

$$I=\frac{140}{A+100} \text{ for pin spans,}$$

$$I+\frac{130}{A+100} \text{ for riveted spans.}$$

For double track bridges, .7 tenths the above values are sufficient.

The common formula for impact is that of C. C. Schneider, now embodied in American Railway Maintenance of Way Specifications:

$$I = \frac{300}{A + 300} S$$

in which as above, I = impact, S = the computed maximum live load stress, and A = the length of loaded track as in the preceding notation.

The deficiences in this formula are thus stated by O. H. Ammann, Trans. Am. Soc. C. E., 1918, p. 911. It gives excessive values for long spans and ballasted bridges and insufficient values for short stringers, on which the dynamic effect is greater than 100 percent.

A detail computation may clear up the relation of deformation where the part is under a dead stress, D, and has suddenly imposed,

If $D = O$, then the dynamic work $Q = Le$ in which e is the elastic deformation due to the load L. But for $D > O$, the piece is under an initial stress D, the dynamic work $Q = \frac{Le}{L+D}$ instead. After vibration has ceased with the load still in place the stored energy which the member can give back in raising the load during gradual unloading is $Le/2$.

7. **Cushioning the Impact of a Striking Body.**—Special problems of this kind are readily solved by proportioning from analogous experience or data.

The kinetic energy in a moving body by mechanics is the product of the mass times the square of the velocity or $Q = (MV)^2$. The cushion effect, as for a saw mill carriage, is to stop a car weighing four tons at a piston speed of 20 miles per hour. A 15″ air cushion is found sufficient to perform this service.

In the Transporter Bridge at Duluth, the weight of the car equals 120 tons; speed 4 miles per hour. The relative kinetic energy of the traveling ferry car is $120 \times (4)^2 = 16 \times 120 = 1920$ tons per hour units. The kinetic energy of the saw mill carriage is $4 \times (20)^2$ in the same units = 1600 similar ton units. Two 15 inch cushions were provided to receive the Ferry car which is nearly twice as liberal a provision as is made in saw mill practice, a provision which would permit the car to run into the slip at full speed and come to rest under the resistance of the cushions without damaging the structure.

8. **Impact in Machines.**—It is difficult and generally impossible to determine the magnitude of the forces and the energy representing

shock which exceeds the effect of a suddenly applied load in machines of various classes subject to hard usage. Such machines must be developed by experience rather than computation, by careful observation of the service which the particular part gives, increasing or diminishing proportions as the service permits, or demands. The original design should be based upon the best available comparison of the endurance of the parts in analogous service.

Where the part subject to shock is able to deflect, as a spring, then the work of flexure is capable of absorbing a large amount of kinetic energy which direct elastic deformation of material cannot do because of the relatively small motion which it permits within the elastic limits of the metal.

When the service is such that the part will be strained by shock beyond the natural elastic limits of hard steel, then medium grade ductile metal may last longer than the material with higher elastic limits. Constant renewals and limited life of the part will result from such a condition. Increase in size within the limits imposed by reasonable bulk and cost may effect a practical or reasonable and economic period of endurance.

9. Working Stresses—Steel.—The uniformity of mild steel of good quality permits the adoption of working stresses approximately as defined by its elastic qualities without allowing that wide margin necessary to cover lack of uniformity and wide divergence in resistance to tension and compression in different directions necessary when using stone, brick and timber. The fact that mild steel has a definite range of thermal proportionality under stress gradually applied that within this range states of residual strain become dissipated and relieved with corresponding improvement in tenacity as measured by the modulus *E* the ductility and reduction of area at rupture, that beyond this limit there is a range of imperfect elasticity within which under range of stress residual strain accumulates with deleterious effect indicates that rational design should employ stresses based thereon.

This limit for soft steel is approximately 22,000 lbs. per square inch and for medium steel (60,000 to 70,000 lbs. per square inch ultimate) 24,000 to 26,000 lbs. per square inch. Three-fourths of these values may be taken for the working stress, to wit; 16,000 bls. per square inch for soft steel and 18,000 lbs. per square inch for medium steel.

Such values allow a thirty percent variation for the limit of inequality of stress distribution which may occur through slight eccentricity of loading, secondary stresses and subsidiary effects disregarded in the general analysis of predominant action.

Impact allowance is to be made for dynamic effect of suddenly applied loads and the structural vibration resulting therefrom. Extreme stresses from wind pressure such as are known to occur at rare intervals from hurricane or cyclonic storms may safely exceed these values twenty five to thirty five percent.

In disregarding secondary stresses, good practice requires if not thorough precise investigation, at least limitation by such truss triangulation that the departure from the equilateral triangle is not too extreme nor the proportions of depth to span length such that relatively excessive deflection results.

STONE: In dealing with stone masonry we have wide variation in compressive strength of the material, relatively small tensile resistance which necessitates careful embedment of the material that it may not crack under flexure.

The strength of the masonry depends on the care with which the stone are quarried and the uniformity with which they are embedded or placed in the mass. The resulting strength is thus a function of the mortar forming the bed or joints, and the stone forming the major portion of the body.

The strength of the wall must depend on the weaker material which is usually the mortar joint, and permissible load carrying capacity is limited more by the mortar joint in rough work than by the compressive resistance of the individual stones.

The strength of the mortar depends on the kind, whether ordinary lime mortar is used or natural or Portland cement mortar employed. Lime mortar hardens slowly, particularly in the interior of a large mass. Natural cement or hydraulic lime is an improvement in this respect though inferor to Portland cement.

Permissible working pressures per square foot have in practical work been arrived at by experience rather than by investigation of the crushing strength of the stone and its characteristics in small samples. Larger blocks show relatively less resistance than the small sample because there is more opportunity for planes of weakness due to upheaval of the material in its natural bed. With the more general introduction of Portland cement, working values used in the past with hydraulic lime and weaker mortars may be logically increased.

COMPRESSIVE STRENGTH OF MORTAR ½ INCH JOINT:

Portland Cement Mortar, 3 mos. old, 1 cement 3 sand	55 tons per sq. ft.
Portland Cement Mortar, 3 mos. old, 1 cement 2 sand	70 tons per sq. ft.
Natural Cement Mortar, 3 mos. old, 1 cement 3 sand	12 tons per sq. ft.
Lime Mortar, (best)	8 to 9 tons per st. ft.

STONE WORK

Rubble walls, lime mortar	5 tons per sq. ft.
Rubble walls, Portland cement mortar	15 tons per sq. ft.

DIMENSION STONE SQUARED IN CEMENT MORTAR

Sandstone and Limestone	20 tons per sq. ft.
Granite	30 to 40 tons per sq. ft.
With ⅜ in. dressed joints	60 to 80 tons per sq. ft.
Granite	40 tons per sq. ft.
Marble, high grade	30 to 40 tons per sq. ft.
Sandstone	25 to 35 tons per sq. ft.

In practice neither stone nor brick work should be trusted with more than 1/8th to 1/12th their crushing load. Stone generally commences to crack at about half the ultimate load. When thoroughly wet some sand stones loose half their strength.

See:—Tables of Crushing Strength of Granite, Lime and Sandstone, given in Lanza's Applied Mechanics, pp. 551-554. J. Wiley 1886. Kidder's Architect's Pocketbook. Trautwine Civil Engineer's Pocketbook, and American Civil Engineers Pocket Book, Merriman.

WORKING STRENGTH OF BRICK WORK ALLOWED BY DIFFERENT BUILDING CODES

Materials	Boston 1909	Buffalo 1909	New York 1906	Chicago 1914	St. Louis 1907	Philadelphia 1914	Denver 1898
	Allowable pressures in tons per sq. ft.						
Hard burned brick in Portland cement mortar	20	12	15	12½	21½	......	......
Hard burned brick in natural cement mortar	18	9	......	10	......	15	9
Hard burned brick in cement and lime mortar	12	......	11½	......	11½	12	......
Hard burned brick in lime mortar	8	6	8	6½	11	8	8
Pressed brick in Portland cement mortar	......	12	......	......	......	......	......
Pressed brick in natural cement mortar	......	9	......	......	......	......	12

Bulletin No. 27 University of Illinois gives a table of results of crushing strength of brick piers 12″×12″, one to three lime mortar, 3 to 10 ft. high, ranging from 8 to 14 percent of the crushing strength of the individual brick, ultimate strength from 80 to 130 tons.

10. Strength of Timber: The test strength of timber depends on the size of the piece. A full size stick which has both heart wood and sap, will give much lower values for the modulus of rupture and compression than a smaller sample containing the heartwood only. If the timber is thoroughly seasoned and dried out, it may readily be 20 to 25 percent stronger than as though it were wet through.

In general the compressive resistance of struts and square timber posts, lengthwise of the grain, is half as great in a large size stick as in smaller samples, the strength of which has been given in various published tables.

From Report of the Association of Railway Superintendents, 1895 the following table of the average strength and safe stresses of various kinds of timber is tabulated.

"See also Cambria Steel Handbook."

AVERAGE ULTIMATE BREAKING UNIT STRESSES, IN POUNDS PER SQUARE INCH

Kind of Timber	Tension		Compression			Transverse		Shearing	
	With Grain	Across Grain	End-Bearing	Columns under 15 diams.	Across Grain	Extreme Fiber stress	Modulus of Elasticity	With Gr.	Across Grain
White Oak	12,000	2,000	7,000	5,000	2,000	7,000	1,500,000	800	4,000
White Pine..	7,000	500	5,500	3,500	700	4,000	1,000,000	400	4,000
Southern Long Leaf or Georgia Pine.......	12,000	600	7,000	5,000	1,400	7,000	1,500,000	600	5,000
Douglas Fir.	8,000	....	5,700	4,500	800	5,000	1,400,000	600	...
Short-leaf Yellow Pine	9,000	500	6,000	4,500	1,000	6,000	1,200,000	400	4,000
Red Pine (Norway pine)	8,000	500	5,000	4,000	800	5,000	1,130,000	....	....
Spruce and Eastern Fir.	8,000	500	6,000	4,000	700	4,000	1,200,000	400	3,000
Hemlock	6,000	-		4,000	600	3,500	900,000	350	2,500
Cypress	6,000	....	5,000	4,000	700	5,000	900,000	....	....
Cedar	7,000	....	5,500	3,500	700	4,000	700,000	400	1,500
Chestnut	8,500	-	-	4,000	900	5,000	1,000,000	600	2,000
California Redwood	7,000	....	....	4,000	600	4,500	700,000	400	...
California Sprue..	....		....	4,000	....	5,000	1,200,000		..

Tests at the Watertown Aresenal show the ultimate crushing strength of white pine posts 5 inches square by 10 feet in length, 2,500 to 3,000 pounds per square inch; 22 feet in length about 1,500 pounds. Yellow pine posts, 12 feet in length, 5 by 5 inches approximately 5,000 pounds per square inch; 20 feet in length of the same dimension, 2,500 to 3,000 pounds per square inch.

The modulus of rupture under transverse stress of 12 by 12 and 6 by 12 timbers, ten to fifteen feet in length, red pine, fir and yellow pine, were found to vary from 4,100 to a little more than 5,000 pounds per square inch.

AVERAGE SAFE ALLOWABLE WORKING UNIT STRESSES, IN POUNDS PER SQUARE INCH

Kind of Timber	Tension		Compression		Transverse		Shearing	
			With Grain					
	With Grain	Across Grain	End Bearing	Columns under 15 Diam.	Across Grain	Extreme Fiber	With Grain	Across Grain
Percent of Ultimate....	10	10	20	20	25	16⅔	25	25
White Oak............	1,200	200	1,400	1,000	500	1,200	200	1,000
White Pine...........	700	50	1,100	700	200	700	100	500
Southern Long-leaf or or Georgia Yellow Pine	1,200	60	1,400	1,000	350	1,200	150	1,250
Douglas Fir...........	800	......	1,100	900	200	800	130	......
Short Leaf Yellow Pine..	900	50	1,200	900	250	1,000	100	1,000
Red Pine (Norway Pine)	800	50	1,000	800	200	800	......	......
Spruce and Eastern Fir..	800	50	1,200	800	200	700	100	750
Hemlock..............	600	......	......	800	150	600	100	600
Cypress...............	600	......	1,000	800	200	800	......	......
Cedar................	700	......	1,100	700	200	700	100	400
Chestnut.............	850	......	......	800	250	800	150	500
California Redwood....	700	......	......	800	150	750	100	......
California Spruce......	......	......	......	800	......	800	......	......

11. Summary.—Heretofore states of strain have been treated as a problem of pure geometric analysis founded upon Hooke's law without complete correlation to the states of stress producing the strain under consideration.

The strain potential of elastic displacement has been arbitrarily assumed equal to the mere external mechanical work of deformation as though independent of thermo dynamic laws and the relations they impose upon the mechanical reactions of elastic deportment.

While Thomson and Tait, and Joule have placed these laws in a clear light, no idea of utilizing them to clear up the complex problems of the engineer appears in the past history of the development of the mathematical theory of elasticity. No investigations of the magnitude of the reactions of residual strain have been made prior to the work of the author and disregard of such effects has in a large measure vitiated conclusions drawn from a large amount of expensive and painstaking experiment.

Conversely, the theory of endurance of materials embodied in fatigue formulas is inharmonious with and contradictory to every day experience and the results of scientific test.

The effort to analyze strain in terms of static modulii renders inexact the geometric relations of E, F, K and B to an extent which even the ingenious introduction of *21 different static coefficients of elasticity fails to clarify.

Failure to properly correlate states of strain with the precise state of stress which causes the strain accounts for the common error of assuming the value of $^*K = .5$ for constant volume. The coefficient of normal contraction to the curves of displacement under axial strain in each of the four quadrants for constant volume is in fact .5 but the geometric coefficient K is nevertheless .4142 as before shown for the resultant elongation.

Eddy, Researches in Graphical Statics, 1878, points out some of the inherent difficulties in the general application of mathematical theory to the problems of engineering mechanics:

> "Internal stress may be defined as the action and reaction of molecular forces. Its treatment by analytic methods is necessarily encumbered by a mass of formulae which is perplexing to any except an expert mathematician. It is necessarily so encumbered, because the treatment consists in a comparison of the stresses acting upon planes in various directions, and such a comparison involves transformation of quadratic functions of two or three variables, so that the final expressions contain such a tedious array of direction cosines that even the mathematician dislikes to employ them
>
> Now, the whole difficulty really lies in the unsuitability of Cartesian coordinates for expressing relations which are dependent upon the parallelogram of forces, and does not lie in the relations themselves, which are quite simple."

The graphical presentation of the subject in Sec. II, will follow the line of attack which Dr. Eddy as early as 1878 conceived to be most practical in the hope that it may thus be placed within the grasp of those who do not possess a high order of mathematical attainment and to render assistance to those who wish to familiarize themselves with the analytical work of the pioneer elasticians Marriotte, Euler, Bernouilli, Navier, Cauchy, Poisson, Kirchoff, Neuman, Klebsch, St. Venant, Green, Kelvin and Rankine.

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*See Art. Elasticity, Ency. Brit.

*See Prof. Deimel's article Elasticity Ency. Americana.

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BY

H. H. EDDY, C. E., Ph. D., Sc. D. L. L. D.

AND

C. A. P. TURNER, C. E.

PART I—BUILDINGS

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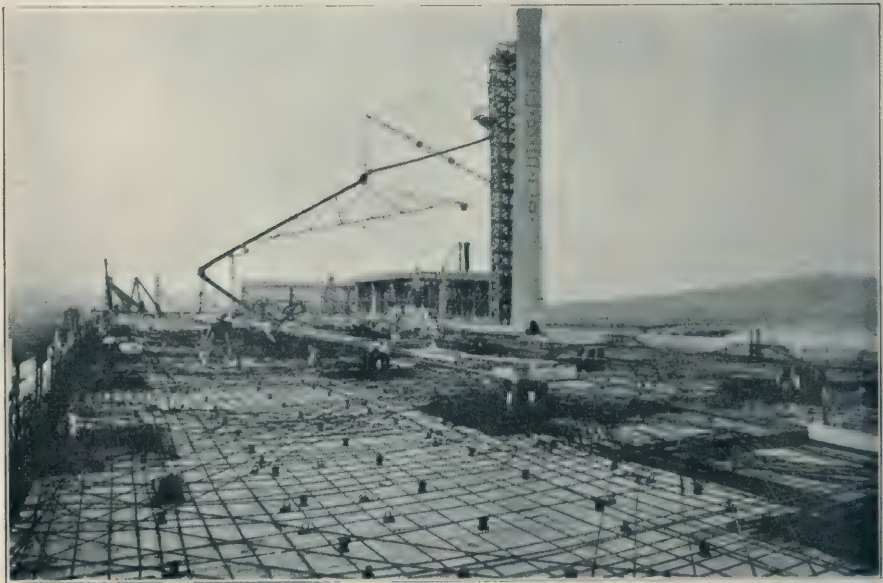

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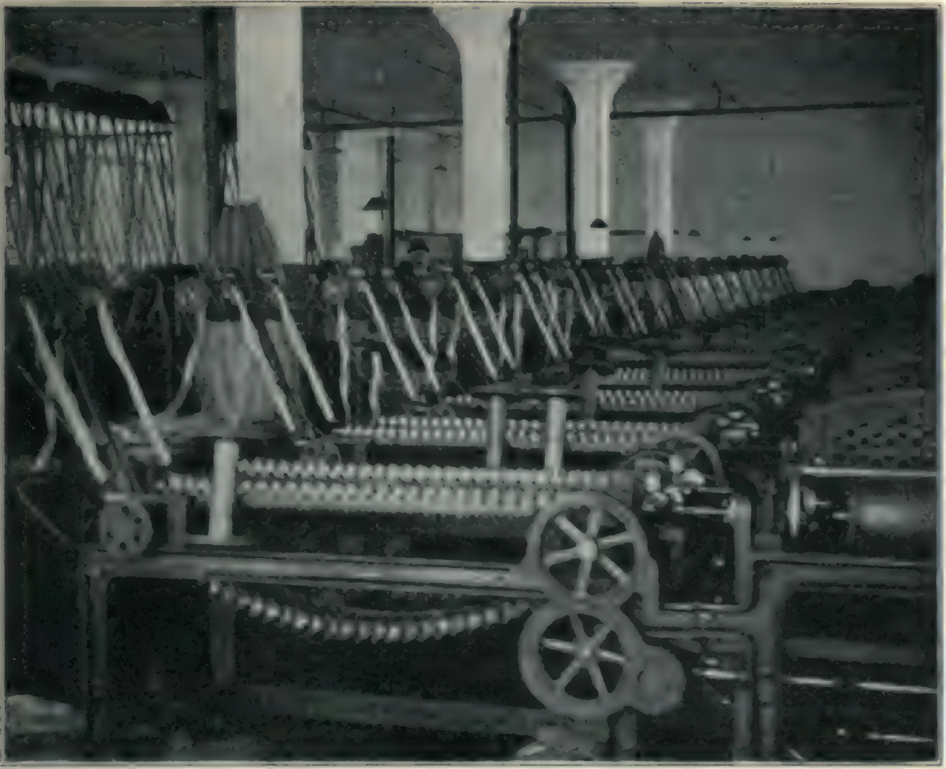

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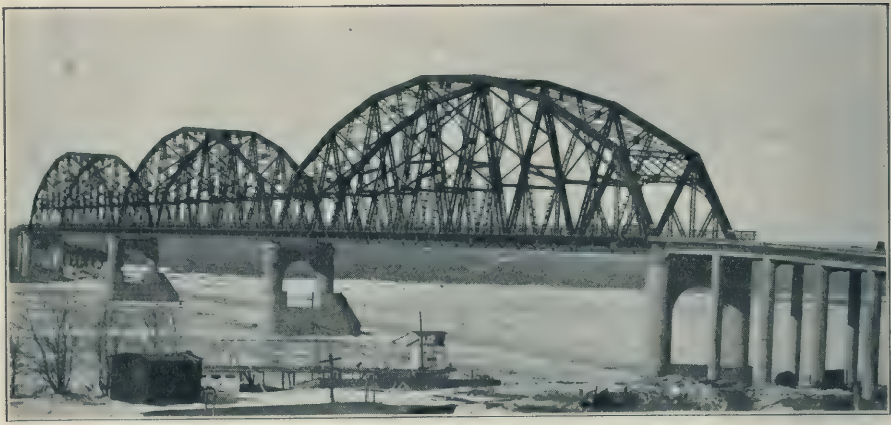

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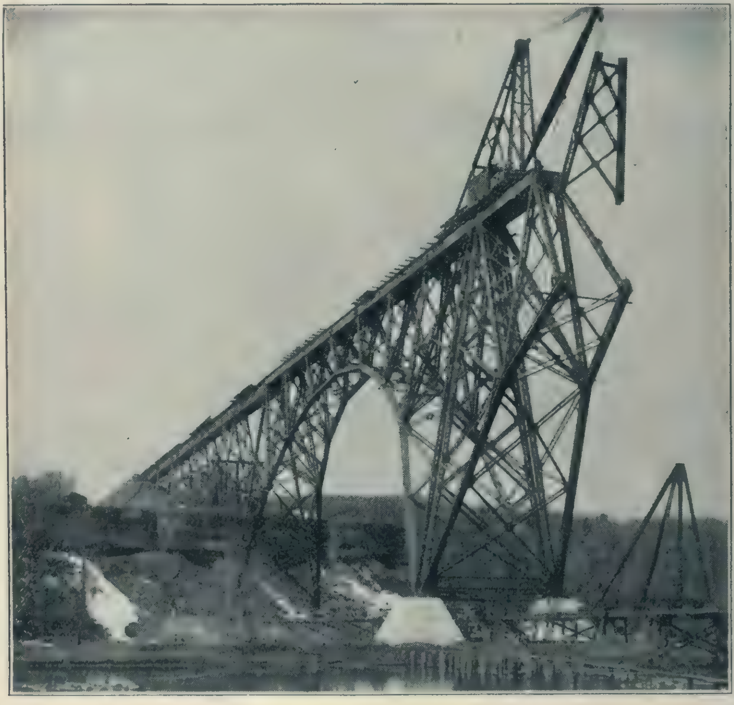

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